

## MISCELLANEA



# **Energy prices forecasting using nonlinear univariate models**

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## **Abstract**

This study analyses whether nonlinear methods are powerful enough to outperform consistently the no-change forecast for prices of key energy commodities, i.e. Brent crude oil, WTI crude oil, natural gas and coal. Six classes of nonlinear models are tested: threshold models (both self-exciting and external threshold variable model approach), smooth transition models (self-exciting and external threshold variable model approach), Markov regime switching models and neural networks. The forecasting competition is designed to simulate a real-time forecasting scheme. The analysis provides some evidence for predictive capabilities of nonlinear methods, but only in short-term horizons.

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**Keywords:** energy commodities, prices forecasting, nonlinear models

**JEL:** C24, C34, L71, Q47

## 1. Introduction

Forecasting commodity prices is one of the most recognized challenges in international economics. The topic is of high interest to both policy and business decision makers. The ability to formulate accurate price forecasts helps to pursue political and economic goals at the country level and also supports the investment and business decision process at the firm level.

The role of oil, gas and coal prices for the real sector of the global economy cannot be overstated. According to International Energy Agency (2017), around one-third of the global energy supply is derived from oil, one-fourth from coal and one-fifth from gas. The energy commodities significantly influence the pace of economic growth in both developed and developing countries as their prices play a dominant role in the industry sector, transportation, cost of electricity and costs of doing business (International Energy Agency 2016).

Additionally, for many developing countries, whose economy is based predominantly on exports of energy commodities, unexpected price movements have a major impact on overall macroeconomic performance. Energy commodity prices are also of paramount importance for the utility sector. Their forecasts are used in commodity trading decisions, electricity production planning and operations activities. Finally, Fattouh, Kilian and Mahadeva (2013) also underline that since the 2000s the popularity of commodities as an asset class for investors has been growing, with oil and gas being the most actively traded commodities. Hence, commodity prices forecasting has also become the area of interest for financial markets participants. The high volatility of these markets often attracts price speculators looking for arbitrage opportunities, especially in critical moments of sudden shortage of a commodity, a hit of excessive supply or political disturbance.

For many years, the literature on commodity prices forecasting was dominated by traditional, linear time series models. However, in recent years both the statistical theory and an increase in computational power have given researchers the opportunity to relax the linearity assumption and try to capture the nonlinear patterns hidden in the commodity prices dynamics. Therefore, the aim of this study is: (i) to understand the price dynamics of the most important energy commodities (oil, gas and coal) and (ii) verify the hypothesis whether using nonlinear methods helps to produce more accurate medium-term forecasts than those from the naïve random walk model,<sup>1</sup> which assume that the best prediction for tomorrow is today's price.

Forecasting energy commodity prices at a reasonable accuracy level is challenging due to many reasons, among others, their high volatility, uncertainty of political decisions influencing the economy or financialization of commodity markets (Bowman, Husain 2004). There is an ongoing debate in the literature as to whether economic or econometric models are able to consistently outperform random walk forecasts (see, e.g. Hamilton 2009; Alquist, Killian, Vigfusson 2013). Although some researchers are skeptical, there are some promising studies presenting predictability evidence for some energy commodities and forecasting horizons (Barsky, Kilian 2002; Chen, Zhuo, Yanping 2015; Issler, Rodrigues, Burjack 2014).

Early studies on forecasting commodity prices concentrated their efforts on historical trend analysis. Lately, however, there have been numerous empirical studies revealing the nonlinear nature of economic and financial data, where linear methods were not able to capture the complex

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<sup>1</sup> Terms “naïve forecasts”, “no-change forecasts” and “random walk forecasts” are used in the study interchangeably. All these expressions are used when we take the current value of the variable as the best forecast for the future. Defining the RW process as  $y_t = y_{t-1} + \tilde{n}_t$  where  $\tilde{n}_t$  is a random term, the best forecast for this model is the naïve/no-change forecast defined as  $y_{t+1}^p = y_t$ .

dynamics. For example Cuaresma, Jumah, Karbuz (2009) outperform the benchmark in out-of-sample oil process forecasting by proposing a simple nonlinear univariate time series model accommodating asymmetric cyclical behaviour. Cashin, McDermott and Scott (2002) and Roberts (2009) show that some commodities behave in asymmetric cycles, where slump phases last longer than boom phases. In addition, some researchers argue that commodity prices exhibit long-lasting cycles or super cycles lasting between 20 and 70 years (see, e.g. Jerrett, Cuddington 2008). All that research presents strong evidence of complex nonlinearities in commodity price behaviour. In particular, a Markov regime switching model, proposed by Hamilton (1989), had many empirical applications, among others, in detecting commodity prices bubbles (see Zhang, Wang 2015). Ahti (2009) extends the range of nonlinear models used to predict commodity prices to threshold and transition autoregression models and neural networks. Some researchers argue that linear models are no longer suitable for describing the complexity of commodity markets. In that sense, Dutta, Bouri and Roubaud (2018) show evidence that the association between oil and gold markets appears to be nonlinear and asymmetric. The nonlinear methods are given a lot of attention in the empirical literature of forecasting stock returns (Rapach, Zhou 2013; White 2006) as well as economic variables (Stock, Watson 1999; Teräsvirta 2006), financial time series (González-Rivera, Lee 2009) or electricity prices (Weron 2014).

As regards energy commodity price forecasting with nonlinear models, the literature in recent years has been boosted by research accommodating the latest artificial intelligence (AI) and machine learning (ML) techniques. A wide range of different methods have been successfully tested on the energy market, from neural networks (NN) of different types and architectures (Movagharnejad et al. 2011; Fan, Wang, Li 2016; Wang, Wang 2016) to support vector machine (SVM) models (Papadimitriou, Gogas, Stathakis 2014). Although AI and ML methods have shown some promising results, there are some studies pointing out their drawbacks, such as likely over-fitting, local minima and dependence on the sample choice and size (Zhang, Zhang, Zhang 2015). Therefore, many researchers concentrate their efforts on finding an optimal forecasting model in the spectrum of more traditional econometric techniques or a hybrid method combining AI and econometric modelling. The latter group is represented by a wide variety of different novel techniques (e.g. Chiroma, Abdulkareem, Herawan 2015; Zhang, Zhang, Zhang 2015; Cheng et al. 2019). However, taking into account the trade-off between in-sample accuracy and out-of-sample variance increase due to model complexity, some researchers try to account for nonlinearity in a more simple way. A recent work of de Albuquerque et al. (2018) shows two interesting results: first their self-exciting threshold autoregressive model (SETAR) outperformed VAR, ARIMA and RW benchmarks both in-sample and out-of-sample for crude oil prices. Secondly, the authors also present an in-sample forecasting accuracy comparison with previous studies, showing that their SETAR model outperforms most of the other models, including those which used ML techniques. The value of accounting for nonlinearity in forecasting crude oil prices is also reported by Wang, Liu and Wu (2017) with a TVP model and by Huang, Yang and Hwang (2009) and Rubaszek et al. (2020) within a threshold framework. However, in the latter work the authors point out that accounting for nonlinearities is significant only in-sample and does not improve out-of-sample forecasts. Similar results are obtained by Rubaszek, Karolak and Kwas (2020) for industrial metals forecasting. In particular, the authors show that introducing nonlinearities within a threshold structure does not have to lead to an improvement in the quality of forecasts. This mixed evidence in the literature leads to the conclusion that the bias-variance dilemma can also be experienced for less complex nonlinear frameworks.

Another advantage of nonlinear models is their ability to account for structural breaks, which are identified for commodity prices in the literature. For example Van Robays (2016) estimates a threshold vector autoregressive (TVAR) model with two regimes to show that the impact of oil shocks differs in times of high and low macroeconomic uncertainty. Similarly, Joëts, Mignon and Razafindrabe (2017) document the influence of macroeconomic uncertainty on oil, but also on 19 other commodity prices, through the estimation of a structural threshold vector autoregressive (TVAR) model. Uddin et al. (2018) use a Markov regime switching model to examine the nonlinear effect of oil price shocks on precious metal returns. The authors find evidence of switching between low and high volatility regimes.

Some studies argue that there is no clear evidence in favour of nonlinear over linear models in terms of forecast performance (De Gooijer, Kumar 1992). In more recent research, Clements, Franses and Swanson (2004) point that adding complexity relative to rival linear models does not usually result in the expected gains in terms of out-of-sample forecast accuracy as the linear approximations give a reasonable forecast accuracy in many empirical studies. On the other hand, there are some reasons to be optimistic about nonlinear models in forecasting as they better reflect data generating process.

This article contributes to the debate by verifying the hypothesis of whether using nonlinear methods helps to outperform no-change forecasts for prices of key energy commodities in a consistent manner. As the empirical results in the literature are ambiguous so far, we believe a comprehensive study including various commodities, different nonlinear models and many data samples would benefit the discussion with a more exhaustive view. For the study we use four main energy commodities, i.e. Brent crude oil, WTI crude oil, natural gas and coal. Six classes of nonlinear models are tested: threshold models (both the self-exciting and external threshold variable model approach), smooth transition models (self-exciting and external threshold variable model approach), Markov regime switching models, and neural networks. The random walk and ARIMA model are used as benchmarks. The forecasting competition is designed to simulate a real-time forecasting scheme. This article presents new results using a comprehensive forecasting competition with a wide variety of models.

The remainder of the paper is organized as follows. Section 2 discusses the models taking part in the forecasting race. Section 3 describes the data and the forecasting competition design. In Section 4 the results of the race are presented. Section 5 contains the conclusions.

## 2. Round-up of forecasting methods

The following methods are considered in the forecasting competition. Benchmarks are random walk (*RW*) and *ARIMA* model.

### 2.1. Threshold autoregression model

Threshold autoregression models (*TAR*) in nonlinear time series analysis were introduced to the literature by Tong (1978). The intuition behind the mechanism of these models is that the data generating process gives a sequence of distinguishable linear autoregressions called regimes. It is the role of a threshold variable to determine the data generating process. A threshold variable can be

simply any variable or the analysed time series itself. Following Franses and van Dijk (2000), a two regime *TAR* ( $p_1, p_2$ ) model with delay  $d$  and autoregressive parameters  $p_1, p_2$  is defined as following:

$$y_t = \left( \phi_{0,1} + \phi_{1,1}y_{t-1} + \dots + \phi_{p_1,1}y_{t-p_1} \right) I[z_t \leq c] + \left( \phi_{0,2} + \phi_{1,2}y_{t-1} + \dots + \phi_{p_2,2}y_{t-p_2} \right) I[z_t > c] + \varepsilon_t \quad (1)$$

where  $z_t$  is a threshold variable and  $c$  is the value of threshold parameter. The order of autoregression in both regimes (denoted by  $p_1$  and  $p_2$ ) may differ.

In one case we assume that  $z_t = y_{t-d}$  so that we apply the model that is called self-exciting threshold autoregression model: *SETAR*( $p_1, p_2, d$ ). In the second case we use assume that the threshold variable describes the deviation of the log price  $x_t$  from its recursive mean  $\bar{x}_t$  lagged by  $d$  periods:

$$z_t = |x_{t-d} - \bar{x}_{t-d}| \quad (2)$$

This specification will later be referred to as the threshold autoregressive model with a mean distance threshold *TARMDT*( $p_1, p_2, d$ ).

For a *TAR* model with an external threshold variable and a given threshold value we additionally assume that the process is in high regime level whenever

$$z_t > c \quad (3)$$

where  $c$  is given as a single standard deviation of the price log-level, so the threshold value is not estimated.

This specification will be later referred to as the threshold autoregressive model with a mean distance exogenous threshold *TARMDET*( $p_1, p_2, d$ ).

## 2.2. Smooth transition autoregression model

Another type of threshold models used in time series analysis and forecasting are smooth transition autoregression models (*STAR*) popularized by Teräsvirta and Anderson (1992). Generally a two regime *STAR*( $p_1, p_2, d$ ) model with delay  $d$  and autoregressive parameters  $p_1, p_2$  is specified as following (Franses, van Dijk 2000):

$$y_t = \left( \phi_{0,1} + \phi_{1,1}y_{t-1} + \dots + \phi_{p_1,1}y_{t-p_1} \right) (1 - G(z_t; \gamma, c)) + \left( \phi_{0,2} + \phi_{1,2}y_{t-1} + \dots + \phi_{p_2,2}y_{t-p_2} \right) G(z_t; \gamma, c) + \varepsilon_t \quad (4)$$

where  $G()$  is a transition function.

In empirical literature a logistic transition function is most commonly used, in which case the model is called logistic smooth transition autoregression (*LSTAR*). Teräsvirta, van Dijk and Medeiros (2005) show that *LSTAR* model can be specified to capture asymmetric behaviour for low and high

values of the transition variable, which is an interesting feature for modelling time series with changing dynamic properties.

Depending on the threshold variable, we specify a self-exciting *LSTAR* model with threshold variable  $z_t = y_{t-d}$  and a model with a threshold variable depending on the distance of the lagged price log-level from its recursive mean:  $z_t = |x_{t-d} - \bar{x}_{t-d}|$ , called *LSTARMDT*( $p_1, p_2, d$ ). For an *LSTAR* model with a mean distance exogenous threshold *LSTARMDET*( $p_1, p_2, d$ ), we assume that a threshold value  $c$  is a single standard deviation of the price log-level.

### 2.3. Autoregressive Markov switching model

The next model considered in the forecasting contest is the autoregressive Markov switching (*MS – AR*) model, which was proposed by Goldfeld and Quandt (1973) and popularized by Hamilton (1989). The idea of this method is that the parameters of the data generation process differ depending on the realization of another, unobservable process. This process, described by the homogenous Markov chain, controls individual structural changes (switches). The *MS – AR* ( $s$ ) model, where  $s$  is the number of lags, is described as follows:

$$y_t = \sum_{k=1}^K S_{tk} \left( \sum_{s=1}^p \phi_{k,s} y_{t-s} + \varepsilon_t \right) \quad (5)$$

where  $S_{tk} = 1$  if the state variable  $Q_t = k$ , and  $S_{tk} = 0$  otherwise.

The distribution of the hidden state process is given by the transition probability matrix, where  $p_{jk} = P(Q_t = k | Q_{t-1} = j)$ . In the forecasting competition we consider a 2-regime model only.

### 2.4. Autoregressive feedforward neural network

A different family of nonlinear models is represented by the autoregressive feedforward neural network (*AR – NN*) model. Although neural networks were originally introduced as classification methods, there are many applications for macroeconomic and other time series forecasting (i.e. Teräsvirta, van Dijk, Medeiros 2005). Kuan and White (1994) present a wide overview of neural networks from an econometric perspective. Assuming a single hidden layer specification with  $D$  hidden units and  $p$  autoregressive lags, the feedforward network *NN* ( $p, 1, D$ ) can be described as follows (Franses, Dijk 2000):

$$y_t = \phi_0 + \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + \sum_{h=1}^{D-1} \beta_h G(\gamma_{0,h}, \dots, \gamma_{p,h}, y_{t-1}, \dots, y_{t-p}) + \varepsilon_t \quad (6)$$

where  $G()$  is an activation function and parameters  $\gamma$  are connection weights between the  $h$ -th hidden unit with all input units, i.e. explanatory variables.

The intuition behind the neural network mechanism is that it can be seen as a universal approximation tool, which means that by using a sufficient number of layers and hidden units one can approximate a large class of functions (Kuan, White 1994).



### 3. Data and forecasting competition design

The prices of the following four key energy commodities are considered in the competition:

- Brent crude oil,
- WTI crude oil,
- natural gas,
- coal.

The variable choice for the competition is dictated to take into account the most important energy resources worldwide. Brent and WTI are two major benchmarks for crude oil pricing globally. Natural gas is considered as it is a suitable clean alternative to fossil fuels and also it is the third main source for primary energy consumption globally. Coal, after crude oil, is the second main energy source and primary source for electricity generation worldwide (International Energy Agency 2017). All data were extracted from Quandl Financial and Economic Database. Table 1 contains a full list of time series used in the study. All nominal price data (expressed in US dollars) are deflated to real values using US CPI as is usually done in the literature. As commodity prices time series are nonstationary, log-difference transformation is used in the model estimation. Along with suggestions in the literature (Kulkarni, Haidar 2009), data used as neural network inputs are also normalized. We use monthly data for energy commodity prices over the period May 1987 to January 2017, which constitute almost 30 years of observations. The sample covers the period after the oil crises in the 1970s and the 1986 price collapse caused by OPEC's political decisions.

The forecasting scheme is recursive and the forecast horizon ranges from 1 to 12 months. For the first vintage we use 15 years of data from the period May 1987 – April 2002 and generate forecasts for the period May 2002 – April 2003. Then we expand the sample by adding one monthly observation so that for the last vintage we use almost 30 years of data from the period May 1987 – January 2016 with forecasts for period February 2016 – January 2017. The out-of-sample forecast performance is assessed with root mean squared error (RMSE). The values of the statistic are reported in ratios in comparison to the no-change forecast (RW), meaning that the values below unity indicate that the model outperforms the benchmark. In order to assess whether the forecasts from the nonlinear model are significantly different from the no-change prediction, we employ the version of Diebold and Mariano (1995) test adjusted for a small sample by Harvey et al. (1997).

### 4. Results

As a first step to understand the dynamics of commodity prices, we analyse their behaviour in time as well as the distributions of returns. Figure 1 presents the dynamic of real energy commodity prices during the period 1987–2017. The dynamics of Brent crude oil and WTI crude oil prices followed a similar path until 2011, with Brent price typically trading at a slight discount, usually within a 5 USD bound, compared to the WTI price, which was reflecting delivery costs to transport Brent oil into the US market and remain competitive to WTI oil. However, in 2011 the prices of the two crude oils began to diverge, mainly due to the gradual depletion of Brent oil in the North Sea, the fracking boom (Kilian 2017), and oversupply of WTI oil in the US combined with storage issues. Eventually, in late 2014, the prices diverged again as production of WTI significantly exceeded the capacity of the oil pipelines

and more expensive rail transportation was introduced (Chen, Zhuo, Yanping 2015). All energy commodities encountered a significant price upsurge during the financial crisis in 2008. Natural gas price suffered from significant peaks also in the end of 2000, in February 2003, and in the end of 2005, some of them caused by severe weather conditions. According to the EIA reports, the price level in February 2003 soared due to an Arctic blast of cold, combined with strong space-heating demand and limited supply alternative.<sup>2</sup> Similarly, the peak around the second half of 2005 was caused by severe hurricanes, which led to the shutting down of many offshore gas platforms and onshore gas processing plants.<sup>3</sup> Sensitivity to weather conditions is pointed in the literature as one of the reasons for high market volatility (see results in Table 2 showing that the volatility of natural gas is higher than other energy commodities). Other reasons for this fact are capacity constraints and sensitivity to the imbalances between demand and supply due to the large capital requirements and significant lead times for production and delivery of energy (Serletis, Shahmoradi 2006). However, in the recent decade its volatility and price level have stabilized below the long-term mean, possibly due to expanding shale gas production, especially in the US. After 2008, the price of coal peaked again in 2016, unlike oil and gas prices, and was driven mainly by China's decision to decrease domestic mining output, resulting in a sudden surge in overseas orders triggering the price peak (Chestney, Gloystein 2017).

Table 2 contains summary statistics showing that all analysed time series are characterized by asymmetric distribution. For oil prices we observe negative values of skewness, which implies that large drops in oil prices occur more often than high rises. Large and positive values of kurtosis for all variables suggest that the distributions are more peaked in the centre compared to normal distribution, resulting in the "heavy tails" phenomenon. The results for the Jarque-Bera test indicate that the null of normal distribution is rejected at the 1% significance level for all the variables. Next, we confirm the evidence of nonlinearity of the time series by conducting a modified BDS nonlinearity test developed by Brock, Dechert and Scheinkmann (1996). The test was originally designed to test for the null hypothesis of independent identical distribution (iid) against non-random chaotic behaviour. However, as shown in many studies, the BDS test has power against a wide range of linear and nonlinear alternatives (for example, see Brock, Hsieh, LeBaron 1991). In order to perform the test against nonlinearity, we filter the data from linear dependencies with log difference transformation and the best fitted ARIMA model chosen based on AIC information criterion. The results of the BDS test presented in Table 3 show that we reject the null hypothesis that residuals of estimated linear models are iid for all commodity prices time series. Following Brock and Sauers (1988), Scheinkman and LeBaron (1989), and Brock et al. (1996), the results suggest that there may be nonlinear structure in the data. Table 2 also contains the results of the test for the number of thresholds in a SETAR model proposed by Hansen (1999). The author uses simulation methods to calculate asymptotic and bootstrap distributions for the test statistic. The null hypothesis of the test assumes a model with one threshold (2 regimes) against the alternative with 2 thresholds (3 regimes). For all variables the  $p$ -value for the test is greater than 0.1, indicating that there is no evidence for null hypothesis rejection. Therefore, the specifications of the regime models in our forecasting competition assume 2 regime models.

The forecasting competition is conducted for six classes of nonlinear models described in Section 2. Models are compared to the main benchmark – the random walk model. Additionally, a simple autoregressive model (ARIMA) is included in the analysis as a secondary benchmark in order

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<sup>2</sup> Source: [https://www.eia.gov/naturalgas/weekly/archivenew\\_ngwu/2003/02\\_27/ngupdate.asp](https://www.eia.gov/naturalgas/weekly/archivenew_ngwu/2003/02_27/ngupdate.asp).

<sup>3</sup> Source: [https://www.eia.gov/naturalgas/weekly/archivenew\\_ngwu/2005/09\\_08/ngupdate.asp](https://www.eia.gov/naturalgas/weekly/archivenew_ngwu/2005/09_08/ngupdate.asp).

to understand what value it brings allowing for nonlinearities compared with a widely used linear model. For each of the 4 commodities and each of the 6 model classes two different specifications are tested. For self-exciting threshold class models the following representation is considered: *SETAR* (1,1,1) with one lag in both the high and low regime and *SETAR* (2,2,2) with two autoregressive parameters in both regimes and two lag delays in the threshold variable. Similarly, for the smooth transition class of models *LSTAR* (1,1,1) and *LSTAR* (2,2,2) are tested. For a threshold and transition model with an external threshold variable we consider one lag specification for the model with a given threshold value *TARMDT* (1,1,1) and *LSTARMDT* (1,1,1) and two lags when the threshold is estimated *TARMDT* (2,2,1) and *LSTARMDT* (2,2,1). The lag choice is consistent with other empirical studies in which usually no more than 3 lags were used (Stock, Watson 1999; Marcellino 2005; Athi 2009). Following Zhang and Wang (2015), where the authors used a two-state first order autoregressive Markov switching model to examine the evolution process of WTI oil price bubbles, the *MS – AR* (1) is tested. Additionally, the second order autoregression *MS – AR* (2) is considered as the alternative specification of a Markov switching class model. The architecture of neural network is designed carefully due to the fact that adding additional hidden units or autoregressive parameters profoundly increases the number of weights to be estimated. Thus, the choice was to estimate one small *NN*, focusing on keeping down the number of weights and a larger one, letting the algorithm better adjust to the data set. Similar approach was suggested by Teräsvirta, van Dijk and Medeiros (2005). The small network architecture *NN* (1,1,4) contains one autoregressive parameter and four hidden units in the hidden layer, giving 17 weights to be estimated. The large network specification *NN* (2,1,6) allows for two autoregressive parameters and six hidden units in the hidden layer, i.e. 31 weights for estimation. In order to ensure the stability of *NN* results, as starting weights are chosen randomly, network estimation is repeated 100 times and then averaged when computing forecasts.

As the main goal is the simulation of real-time forecasting,<sup>4</sup> we follow Stock and Watson (1999) and re-estimate the models each time an observation is added to the data set, meaning that each specification for every commodity was estimated 166 times to evaluate the forecast at each point of time beginning from May 2002. In total there were  $4 \times 13 \times 166$  models calculated.

Tables 3–5 contain forecast accuracy results for crude oil, natural gas and coal prices respectively. We present RMSE results for RW and ratios for each model in comparison to the RW benchmark. Generally, for all commodities, the results suggest that no model dominates the others and the model performing best is not the same for all commodities and forecast horizons. Although the results show some evidence in favour of nonlinear models predictions against the no-change forecast, it applies only for short horizons and usually is not statistically significant based on the modified DM test.

The results for Brent and WTI crude oil prices coincide with the general remark that there is some evidence in favour of nonlinear models in the short-term forecasting horizon, up to three-steps ahead. The model performing best for Brent crude oil is *TARMDT* (2,2,1), giving on average 6%, 4% and 2% forecast improvement over the naïve forecast in the first three forecasting horizons respectively. In the case of WTI crude oil, for one-step ahead forecasts most specifications of threshold, transition and the Markov regime switching model beats the no-change forecast; however the biggest gain of a 4% decrease in average error can be achieved with the Markov regime switching model *MS – AR* (1). For two-step ahead forecasts the ARIMA model proves to be better. None of the nonlinear models helped to build more accurate forecasts in the medium-term horizon.

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<sup>4</sup> Data revision is not taken into account here.

For natural gas there is almost no evidence in favour of nonlinear models. Only the threshold model with an external threshold variable and a given threshold value outperforms RW in medium-term horizons 9–12 months. However, the difference was not proven to be statistically significant.

The results show that obtaining consistent enhancement in a predictive accuracy for natural gas forecasts is challenging. The question arising when reporting such results is the market efficiency issue. The efficient market hypothesis (EMH) assumes that all new information is reflected instantly in commodity prices so that price patterns are random and no model based solely on past market behaviour can do any better. However, the empirical evidence in the literature is quite diverse. Walls (1995) claims that the EMH for natural gas prices can be rejected in only 3 of the 13 spot markets examined. Chinn and Coibion (2010) report mixed results: for 3-month futures they reject EMH for natural gas, but not for longer futures contracts. On the other hand, many other researchers doubt the efficient-market hypothesis for natural gas prices markets (see Herbert, Kreil 1996 or Arouri et al. 2013). The study of Murry and Zhu (2004), showing that the efficiency of the market might be affected by changing the institutional landscape, adds even more confusion to this picture. In particular, in their study they showed that the introduction and demise of EnronOnline, a commodity trading platform, coincided with the improvement and worsening in the degree of market informational efficiency.

As the literature does not give an unequivocal answer to the EMH question, another reason for the unsuccessful forecasting of natural gas prices may be the fact that natural gas prices are highly sensitive to changing weather conditions and supply disruptions, making price spikes unpredictable in other than day-to-day forecasts. Similar conclusions are reported in the literature. Mishra and Smyth (2016) found that none of their models forecasting natural gas spot prices based on natural gas futures prices or spot-future spread perform any better than the benchmark random walk model. Čeperić, Žiković and Čeperić (2017) recorded only slight improvements over the univariate time series approaches and naïve forecast by incorporating additional variables into the model, including weather indices. Salehnia et al. (2013) tested several nonlinear models for understanding natural gas price dynamics, concluding that the prediction of prices of the best performing model is below expectations and quite noisy.<sup>5</sup>

Coal forecasts obtained with the threshold, transition and MS model outperform RW in one-, two- and three-step ahead forecasts. Some of the differences are statistically significant, i.e. for a threshold model with external threshold variable and a given threshold, *TARMDT* (1,1,1). The model performing best for coal forecasts is the self-exciting threshold model *SETAR* (1,1,1), with a 4%, 5% and 4% forecast error decrease in one-, two- and three-step ahead forecasts respectively.

To summarize the results of the study, Table 6 presents a subset of the outcomes for models, commodities and forecasting horizons that benefit from allowing for nonlinearities, i.e. the average forecast error is lower for the nonlinear model than RW. In general, nonlinear models prove to benefit forecasting results in the short term (typically from 1 to 3 months), with the exception of natural gas, where there is weak evidence of a nonlinear impact in the medium term (9- and 12-month forecast). Comparing the outcomes presented in Table 6, the most consistency in model performance (outperforming RW) is observed for the threshold model *SETAR* (1,1,1) and models with mean distance threshold. The positive results of the latter specification suggests that oil and coal prices (weak evidence for natural gas) might follow a two-regime data generating process where within a certain bandwidth of the mean value the prices behave differently than when they exceed it. Table 6 does not contain

<sup>5</sup> The authors did not report a direct comparison of their models to any standard benchmark like random walk or ARIMA.

a large specification of the neural network model described as  $NN(2,1,6)$ , because it does not provide better results for any commodity.

To draw final conclusions, the overall outcome of the study is presented in the form of a best performing model summary for each commodity and forecasting horizon in Table 7. Nonlinear models outperform RW and the linear autoregressive model for each commodity, however, not consistently in various forecasting horizons. For Brent oil, natural gas, and coal, the best performing models which outpaced RW were the threshold models and for oil the WTI Markov-switching and linear models.

The results presented in this study are not entirely in line with the conclusions from similar research. In particular, the predictive power of the neural network framework, presented in some empirical studies for economic and financial time series (see Marcellino 2005), is not revealed in this model application. Teräsvirta, van Dijk and Medeiros (2005) argue that careful specification of nonlinear time series models is of crucial importance. Therefore, focusing on tuning the model specifications for each of the time series could be worth further research. On the other hand, the poor performance of  $NN$  might be caused by an insufficient number of data points compared with the number of weights to be estimated, resulting in the estimation error being bigger than the possible gain of better model fit. Many positive results from the literature showing the superior predictive power of  $NN$  are based on daily weekly data, where this issue no longer occurs (see, e.g. Shambora, Rossiter 2007; Fan, Wang, Li 2016).

## 5. Conclusions

The study presents an analysis of the dynamics of prices for the most important energy commodities: crude oil, natural gas, and coal. Using monthly data from the period of almost 30 years from May 1987 to January 2017, we performed a forecasting competition search for the consistently best forecasting methods. We applied a linear  $ARMA$  model and 12 different nonlinear models: threshold, transition, and Markov switching models as well as neural networks to compare the forecast accuracy for the prices of key energy commodities.

The results show some evidence that nonlinear methods can outperform the RW model in short-horizon predictions for Brent oil, WTI oil and coal prices, and in the medium-term for natural gas; however, only for Brent oil and coal was the difference statistically significant. Point forecasts statistics indicate that threshold and transition models with a threshold variable based on the distance of the lagged price log-level to the price mean had the best chances to significantly beat the RW model. However, for coal forecasts self-exciting specifications of threshold and transition models and the Markov regime switching model also outperformed the naïve forecast in the short-term horizon. Nevertheless, none of the tested nonlinear methods could consistently outperform the no-change forecast.

This study shows that although outperforming the RW forecasts is a difficult task, some nonlinear methods might be able to help. Of course, there is much to be done in the areas of model estimation and specification techniques, but we can remain optimistic about the relevant role of nonlinear models in forecasting applications.



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## Appendix

Table 1  
Commodity prices – time series description

Commodity	Description	Quandl source code
Brent oil	Crude oil prices: Brent – Europe, USD per Barrel not seasonally adjusted	FRED/MCOILBRETEU
WTI oil	Crude oil, West Texas Intermediate (WTI) 40' API, f.o.b. Midland Texas, spot price crude oil price, WTI, UDS/BBL	COM/WLD_CRUDE_WTI
Gas	Natural gas (US), spot price at Henry Hub, Louisiana Natural gas price, US, USD/MMBtu	COM/WLD_NGAS_US
Coal	Coal, Australian thermal coal, 12,000 – btu/pound, less than 1% sulfur, 14% ash, FOB Newcastle/Port Kembla, USD per metric ton	ODA/PCOALAU_USD
CPI US	Consumer Price Index – USA	RATEINF/CPI_USA

Table 2  
Summary statistics for log-differences of prices and p-values for BDS nonlinearity test

	Mean	Median	Variance	Kurtosis	Skewness	Normality test <sup>a</sup>	BDS test p-value <sup>b</sup>	Regime no. test p-value <sup>c</sup>
Brent oil	0.001	0.004	0.008	2.218	-0.114	76.35***	0.000	0.15
WTI oil	0.001	0.006	0.007	1.801	-0.239	53.43***	0.001	0.63
Gas	0.000	-0.003	0.017	1.198	0.033	22.40***	0.011	0.82
Coal	0.001	-0.002	0.003	7.997	0.368	979.46***	0.006	0.44

<sup>a</sup> Jarque-Bera test of normality. The numbers in the table reflect the value of the Chi-squared statistic and \*\*\* indicates statistical significance at the 1% level.

<sup>b</sup> For each time series the best fitted ARIMA model was chosen based on the AIC information criterion. For Brent oil and WTI oil ARIMA(0,0,1)(0,0,2) was chosen, for gas ARIMA(1,0,2) and for coal ARIMA(3,0,2).

<sup>c</sup> Hansen (1999) test for number of regimes in SETAR model. Null hypothesis states for 1 threshold model against the alternative hypothesis – 2 threshold model.

Table 3

Root mean squared forecast errors from nonlinear models in relation to the no-change forecast for Brent crude oil and WTI crude oil price levels

Oil Brent						
<b>h</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>6</b>	<b>9</b>	<b>12</b>
RW RMSE	0.090	0.143	0.186	0.273	0.322	0.355
ARIMA (1,0,1)	0.984	<b>0.994*</b>	<b>0.999**</b>	1.011***	1.021***	1.021***
SETAR (2,2,2)	0.997	1.013	1.030	1.045	1.039	1.046
SETAR (1,1,1)	0.993	1.000	1.010***	1.036***	1.068***	1.095***
LSTAR (2,2,2)	1.004	1.025	1.038	1.051	1.049	1.046
LSTAR (1,1,1)	0.991	1.006	1.013	1.039	1.056	1.058
MS – AR (1)	0.978	1.039	1.105	1.217	1.417	1.544
MS – AR (2)	0.990	1.110	1.231	1.608*	1.971**	2.304**
NN (2,1,6)	1.058	1.049	1.098	1.271	1.355	1.336
NN (1,1,4)	0.996	1.007	1.002	1.034	1.067	1.074
TARMDT (2,2,1)	0.944	<b>0.958*</b>	0.978	1.016	1.040	1.054
TARMDT (1,1,1)	0.991	1.007	1.024	1.058	1.096	1.126
LSTARMDT (2,2,1)	0.951	<b>0.967*</b>	<b>0.987*</b>	1.035	1.065	1.089
LSTARMDT (1,1,1)	0.992	1.002	1.021	1.077	1.134	1.191
Oil WTI						
<b>h</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>6</b>	<b>9</b>	<b>12</b>
RW RMSE	0.088	0.142	0.188	0.275	0.321	0.349
ARIMA (1,0,1)	0.964	0.976	0.998	1.016*	1.028***	1.026***
SETAR (2,2,2)	0.990	1.005	1.068	1.177	1.240	1.305
SETAR (1,1,1)	0.969	0.985	1.024**	1.086***	1.152***	1.220***
LSTAR (2,2,2)	0.983	1.032	1.130	1.478	2.019	3.157
LSTAR (1,1,1)	1.492	1.225	1.100	1.044**	1.073*	1.085*
MS – AR (1)	0.963	1.032	1.130	1.308	1.515	1.660
MS – AR (2)	0.964	0.996	1.083	1.333	1.625	1.868
NN (2,1,6)	1.020	1.043	1.142	1.368	1.575	1.732
NN (1,1,4)	1.151	1.267	1.450	1.878	2.247	2.640
TARMDT (2,2,1)	0.968	0.986	1.021	1.120	1.209	1.312
TARMDT (1,1,1)	0.977	0.992	1.019	1.058	1.100	1.132
LSTARMDT (2,2,1)	0.969	0.980	1.009	1.094	1.180	1.312
LSTARMDT (1,1,1)	0.969	0.997	1.022	1.052	1.075	1.073

Note: \*\*\*, \*\* and \* denote, respectively, the 1%, 5% and 10% significance levels of the two-tailed modified Diebold-Mariano test.

Table 4

Root mean squared forecast errors from nonlinear models in relation to the no-change forecast for natural gas price levels

Natural gas						
h	1	2	3	6	9	12
RW RMSE	0.127	0.183	0.225	0.321	0.384	0.403
ARIMA (1,0,1)	1.016	1.010**	1.011***	1.014***	1.026***	1.033**
SETAR (2,2,2)	1.049	1.022	1.022	1.024*	1.067**	1.115*
SETAR (1,1,1)	1.016	1.019	1.024**	1.039***	1.097***	1.148***
LSTAR (2,2,2)	1.039	1.031	1.019**	1.032***	1.062***	1.098***
LSTAR (1,1,1)	1.019	1.013**	1.018***	1.024***	1.054***	1.081***
MS – AR (1)	1.006	1.007	1.033	1.087	1.206	1.328
MS – AR (2)	1.048	1.028	1.078	1.150	1.301	1.457
NN (2,1,6)	1.135	1.123	1.140	1.154***	1.170***	1.221***
NN (1,1,4)	1.092	1.086	1.064	1.092*	1.097***	1.084***
TARMDT (2,2,1)	1.016	1.010**	1.011***	1.014***	1.026***	1.033**
TARMDT (1,1,1)	1.048*	1.070*	1.076*	1.015	0.996	0.957
LSTARMDT (2,2,1)	1.049	1.033***	1.037***	1.032**	1.035**	1.038**
LSTARMDT (1,1,1)	1.101***	1.122*	1.123	1.154	1.220	1.278

Note: \*\*\*, \*\* and \* denote, respectively, the 1%, 5% and 10% significance levels of the two-tailed modified Diebold-Mariano test.

Table 5

Root mean squared forecast errors from nonlinear models in relation to the no-change forecast for coal price levels

Coal						
<b>h</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>6</b>	<b>9</b>	<b>12</b>
RW RMSE	0.071	0.116	0.156	0.252	0.330	0.380
ARIMA (1,0,1)	0.972	1.000	1.026	1.078	1.112	1.113
SETAR (2,2,2)	<b>0.982**</b>	<b>0.999**</b>	0.996	1.031	1.077	1.090
SETAR (1,1,1)	<b>0.958**</b>	0.949	0.961	1.032	1.087	1.111
LSTAR (2,2,2)	0.993	1.009	1.058	1.306	1.950	3.519
LSTAR (1,1,1)	<b>0.964*</b>	0.973	0.980	1.012	1.032	1.040*
MS – AR (1)	<b>0.964*</b>	0.960	1.000	1.065	1.160	1.248
MS – AR (2)	0.975	0.988	1.017	1.098	1.199	1.294*
NN (2,1,6)	1.127	1.220	1.312	1.552**	1.741***	1.867***
NN (1,1,4)	1.016	1.068	1.112**	1.241**	1.446**	1.662*
TARMDT (2,2,1)	0.972	1.000	1.026	1.078	1.112	1.113
TARMDT (1,1,1)	<b>0.986**</b>	<b>0.990**</b>	<b>0.996**</b>	1.030***	1.067***	1.087***
LSTARMDT (2,2,1)	0.987	0.994	1.003	1.035	1.068	1.080
LSTARMDT (1,1,1)	1.053**	1.063*	1.081*	1.123*	1.179**	1.212*

Note: \*\*\*, \*\* and \* denote, respectively, the 1%, 5% and 10% significance levels of the two-tailed modified Diebold-Mariano test.

Table 6

Subset of the forecasting competition results for nonlinear models and commodities that outperformed random walk forecasts

Model	h	Oil brent	Oil WTI	Coal	Model	h	Oil brent	Oil WTI	Coal	Natural gas	
SETAR (2,2,2)	1	0.997	0.990	<b>0.982**</b>	NN (1,1,4)	1	0.996				
	2			<b>0.999**</b>		1	0.944	0.968	0.972		
	3			0.996	TARMDT (2,2,1)	2	<b>0.958*</b>	0.986			
SETAR (1,1,1)	1	0.993	0.969	<b>0.958**</b>		3	0.978				
	2		0.985	0.949		1	0.991	0.977	<b>0.986**</b>		
	3			0.961		2		0.992	<b>0.990**</b>		
LSTAR (2,2,2)	1		0.983	0.993	TARMDT (1,1,1)	3			<b>0.996**</b>		
LSTAR (1,1,1)	1	0.991		<b>0.964*</b>		9				0.996	
	2			0.973		12					0.957
	3			0.980							
MS-AR (1)	1	0.978	0.963	<b>0.964*</b>	LSTARMDT (2,2,1)	1	0.951	0.969	0.987		
	2			0.960		2	<b>0.967*</b>	0.98	0.994		
MS-AR (2)	1	0.990	0.964	0.975	LSTARMDT (1,1,1)	3	<b>0.987*</b>				
	2		0.996	0.988		1	0.992	0.969			
						2		0.997			

Notes:

The table contains relative root mean squared forecast errors from a nonlinear model to RW model.

\*\*\*, \*\* and \* denote, respectively, the 1%, 5% and 10% significance levels of the two-tailed modified Diebold-Mariano test.

Table 7

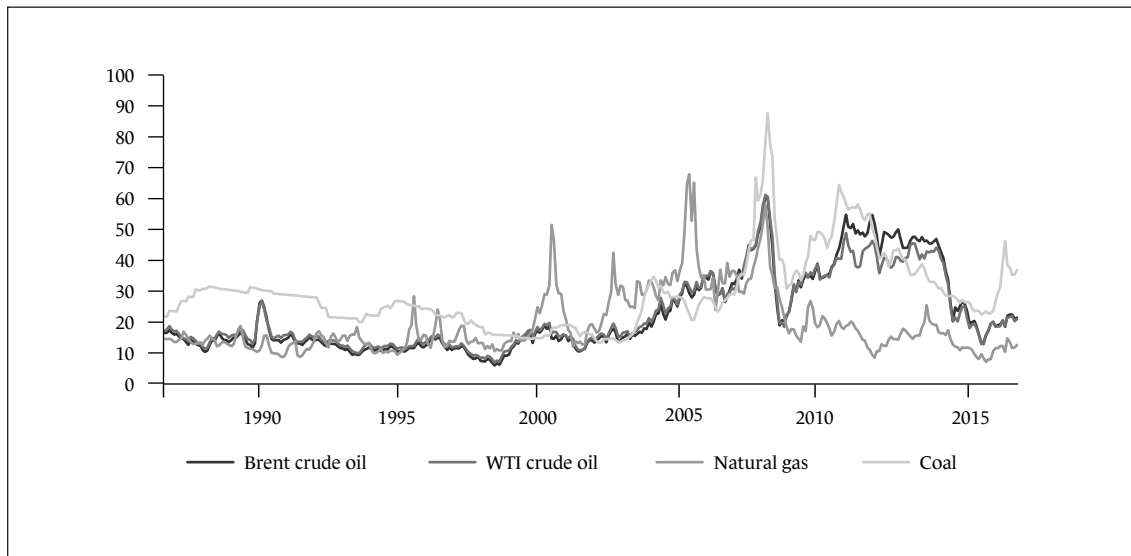
Best performing model for each of the commodity prices analysed in the study

	H1	H2	H3	H6	H9	H12
Oil brent	TARMDT (2,2,1)	TARMDT (2,2,1)	TARMDT (2,2,1)	RW	RW	RW
Oil WTI	MS-AR (1)	ARIMA (1,0,1)	ARIMA (1,0,1)	RW	RW	RW
Natural gas	RW	RW	RW	RW	TARMDT (1,1,1)	TARMDT (1,1,1)
Coal	SETAR (1,1,1)	SETAR (1,1,1)	SETAR (1,1,1)	RW	RW	RW

Note: model forecasting performance is measured in terms of the root mean square forecasting error.

Figure 1

The dynamic of real energy commodity prices during the period 1987–2017 deflated by US CPI



Note: oil prices are expressed in USD per barrel, natural gas in USD per 10 MMBtu and coal in USD per metric ton.

