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Efficiency and stability of transaction systems based on 9 types of moving averages on the example of 140 components of 3 different Warsaw Stock Exchange indexes

Krzysztof Borowski*

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Abstract

Most of the papers assessing the effectiveness of transactional systems focus on optimizing parameters for one financial instrument. There is a visible lack of research covering the problem of the effectiveness of transactional systems using moving averages in a wider frame, i.e. determining which of the moving averages statistically generates the best buy and sell signals. This issue was addressed in this article. Therefore, the purpose of the paper is to determine the effectiveness of transaction systems based on 9 different types of moving averages for companies listed on the Warsaw Stock Exchange, included in the three stock indexes: WIG20 (blue chips), mWIG40 (middle capitalization) and sWIG80 (small capitalization), e.g. for 140 companies in total. The effectiveness of transactional systems based on the intersection of the moving average and closing price was tested for 9 different types of moving averages (exponential, simple, time series, triangular, variable, volume adjusted, weighted, Hull and fractal adaptive). The results clearly showed that in the case of two transactional systems based on two different moving averages, it is possible to indicate the one that for a larger number of companies created a more effective system. On this basis, efficiency rankings for analysed types of moving averages were constructed. In the performance ratings of the individual moving averages (R1 and R2), the moving averages classified at the highest positions were the following: time series and triangular, while the following moving averages were classified at the lowest: fractal adaptive, Hull and variable. In the second part of the paper, the stability of transaction systems for individual types of moving averages was assessed. The trading systems stability rankings proved that according to measures MM1 and MM2 the most stable resulted to be systems based on averages: weighted, exponential and triangular, while the most unstable were those based on the following ways of weighting prices: time series and Hull. Therefore, systems based on time series moving average generate the highest rates of return, but at the same time prove to be the least stable.

Keywords: technical analysis, moving average, transaction systems

JEL: G15, G17

* Warsaw School of Economics; e-mail: krzysztof.borowski@gmail.com.

1 Introduction

Moving averages have been used in technical analysis since the 1960s, although their roots date back to 1901. Originally, they were applied to determine the moment of buying and selling certain assets. The buy signal occurs when the price crosses its moving average above. In turn, the sales signal is generated when the price crosses its moving average down. Then moving averages started to be used as moving levels of support and price resistance, which were originally adapted to determine the range of rising or falling waves. Another application of moving averages was the construction of price envelopes, created as a result of adding or subtracting a specific value of the moving average (e.g. 10%). This type of offset is called the vertical translation of the average. Further modifications led to displace the moving average forward or backward by several sessions (time translation or horizon translation). Translated moving average crosses its price and thus generates buy or sell signals in slightly different moments of time and at different price levels, unlike the intersections of unshifted moving averages and their prices.

Sklarew (1980) proposed applying a forward or backward shift of the moving average by the number of days equal to the square root of the moving average length, rounding the obtained result upwards to the whole number. Thus, moving averages of 2 to 4 sessions are shifted by 2 days, with lengths from 5 to 9 sessions by 3 sessions, with length of 10 to 16 sessions, by 4 sessions, etc.

Bollinger bands, Starc bands and Keltner channels, technical analysis tools, are also calculated on the basis of moving averages (Keltner 1960). Still another method of using moving averages was proposed by Japanese investors. According to them, purchase orders are generated when the N_1 session moving average crosses up the moving *N2* session (with the condition that $N_1 < N_2$). The sales signal is observed when the N_1 session moving average crosses down the N_2 session average. These indications are called the golden cross and death cross, respectively. The above considerations allow to determine the following degrees of freedom of any moving average:

1. The price, which is the basis for the moving average calculation (in most cases it is the closing price for the observed time interval but in others opening prices, the highest or lowest prices or price combinations are also used).

2. The method of calculating the average (simple, linearly weighted, exponentially weighted, etc. – this issue will be discussed later in the paper).

3. Average length of averaging, i.e. time interval, the value of the moving average is calculated, e.g. from 10 sessions or 15 sessions.

4. Vertical shift amount by *X*% of the average value (up or down).

5. The amount of the horizontal shift by *N* time intervals (e.g. by 5 sessions), forward or backward.

In the course of further works on moving averages, transaction systems based on the intersection of three moving averages, e.g. 4, 9 and 18 session (on the stock market) or 5, 10 and 20 session on the commodity market were also introduced. Also double and triple smoothing of the moving average were implemented (the second average is calculated on the basis of the first and the third using the second one).

The most important difference between the various types of moving averages is the weight assigned to the most recent data. Once this “weighting” scheme has been determined, it is held static over the range of calculation. The exception are the variable moving averages and volume adjusted moving average. The variable moving average automatically adjusts its weighting based on market conditions. A variable moving average becomes more sensitive to recent data as volatility increases and less sensitive to recent data as volatility decreases. Similarly, the volume adjusted moving average automatically adjusts as the security's volume increases and decreases. Information on how to calculate the nine types of moving averages is provided below (A description of the moving averages has been made for the daily time interval).

The following convention was adopted later in this paper, according to which $X(C, N)$ denotes the following moving average:

X – type of moving average (simple, exponential, etc. ...),

C – type of argument for which the average value is calculated (e.g. C – closing prices),

N – length of a moving average (e.g. 5 sessions).

The calculations for the following moving averages were proceeded in this paper.

Exponential (*E*)

An exponential (or exponentially weighted) moving average is calculated by applying a percentage of the last closing price to the previous moving average value (Kaufman 1978, pp. 64–66):

$$EMA_t(C,N) = C_t * \left(\frac{S}{N+1}\right) + EMA_{t-1}(C,N) * \left(1 - \frac{S}{N+1}\right)$$

where:

$EMA_t(C, N)$ – exponential moving average for the period t, calculated for closing prices C,

C_t – closing price for the period t,

N – length of a moving average,

S – smoothing parameter.

The method used to calculate an exponential moving average puts more weight toward recent data and less weight toward past data than the simple moving average method does.

Simple (*S*)

A simple arithmetic moving average is calculated by adding the closing price of an asset for a number of time periods and then dividing this total by the number of time periods. The result is the average price of the security over the time period (Kaufman 1978, pp. 58–59):

$$SMA_t(C,N) = \frac{C_t + C_{t-1} + \ldots + C_{t-N+1}}{N}$$

Time series (*T*)

The time series moving average is calculated using linear regression technique. Rather than plotting a straight linear regression line, a time series moving average plots the last point of the line. It does this using the specified number of periods of each day. The individual points are then connected together with a line to form a time series moving average. This moving average is sometimes referred to as a "moving linear regression" study or a "regression oscillator" (Metastock for Windows 2000, p. 460).

Triangular (*Tri*)
A triangular moving average is similar to an exponential and weighted moving average except a different weighting scheme is used. Simple moving averages assign the weight equally across all the data, while exponential and weighted moving averages assign the majority of the weight to the most recent data. With a triangular moving average, the majority of the weight is assigned to the middle portion of the data. To implement triangular weighting, one should begin with the standard formula for a weighted average under the condition that the weighting will increase from 1 to the middle of the window (at $N/2$), then decrease to the end at N. This procedure has a slightly different form when the period (N) is odd or even. A triangular moving average can be approximated with the use of a double-smoothed simple moving average (Kaufman 2013, pp. 289–290; Payne 1989).

Variable (*Var*)
A variable moving average is an exponential moving average that automatically adjusts the smoothing constant based on the volatility of the data series. The more volatile the data, the larger the smoothing constant used in the moving average calculations. The larger the smoothing constant, the more weight given to the current data. The opposite is true for the less volatile data.

Typical moving averages suffer from the inability to compensate for changes in volatility. By automatically adjusting the smoothing constant, a variable moving average is able to adjust its sensitivity, allowing it to perform better in both high and low volatility markets (Thusar 1992):

$$VMA_t(C,N) = (0.78 * VI * C_t) + (1 - 0.78) * VI * VMA_{t-1}(C,N)$$

where VI – volatility index.

Volume adjusted (*Vol*)
Dick Arms, the developer of the Arms Index and the equivolume charting method, presented a unique method for calculating moving averages, which incorporates volume and is appropriately called a volume adjusted moving average (Arms 1987).

The calculation method incorporates volume and is a little complex (Metastock for Windows 2000, p. 461–464):

1) calculating the average volume with the use of every time period in the chart,

2) calculating the volume increment by multiplying the average volume by 0.67,

3) calculating each period's volume ratio by dividing each period's actual volume by the volume increment,

4) starting at the most recent time period and working backwards, multiply each period's price by the period's volume ratio and cumulatively sum these values until the user-specified number of volume increments is reached. So, only a fraction of the last period's volume will likely be used.

Weighted (*W*)
This type of moving average is designed to put more weight on recent data and less weight on past data (Kaufman 2013, pp. 287–289):

$$WMA_t(C,N)=\frac{C_{t-N+1}*1+C_{t-N+2}*2+\ldots+C_t*N}{1+2+\ldots+N}$$

Two less commonly used moving averages are the moving Hull and fractal.

Hull moving average (*Hull*)

The Hull moving average (HMA), developed by Alan Hull, is an extremely fast and smooth moving average. In fact, the HMA almost eliminates lag altogether and manages to improve smoothing at the same time. The way of calculating the Hull moving average can be described as follows (Gardner 2010; Kaufman 2013, pp. 306–307):

1) calculate a weighted moving average with period N/2 and multiply it by 2,
2) calculate a weighted moving average for period *N* and subtract if from step 1,
3) calculate a weighted moving average with period square root (sqrt(*N*)) using the data from step 2:

$$HMA_t(C,N)=WMA_t\left(2*WMA_t\left(C,\frac{N}{2}\right)-WMA_t(C,N),\sqrt{N}\right)$$

Fractal adaptive moving average (FRAMA)

This type of moving average was developed by John Ehlers with the use of the algorithm of the exponential moving average, in which the smoothing factor is calculated based on the current fractal dimension of the price series (Ehlers 2005):

$$FRAMA(C,t)=A_t*C_t+(1-A_t)*FRAMA(C,t-1)$$

where:

$FRAMA(C, t)$ – current value of FRAMA,
A_t – current factor of exponential smoothing.

Exponential smoothing factor is calculated according to the formula below:

$$A_t=e^{-4.6*(D_t-1)}$$

where D_t – current fractal dimension.

The advantage of FRAMA is the possibility to follow strong trend movements and to sufficient slow down at the moments of price consolidation.

Each of the above-described moving averages is characterized by its advantages and disadvantages. In the case of average *S*, all prices in the considered time interval have the same weight. So in the case of the 15 session moving average, the price from 3 weeks ago is as important as the last price. This fact contradicts the approach expressed by investors, for whom the last price is definitely more important than the first in a given time horizon. Therefore, in their opinion, the moving average *W* and *E* represent an approach in line with investors' expectations. In the case of the average *W*, the price weight in the averaging window decreases linearly, which is not consistent with the theory of behavioural finance, according to which the exponential change in weight in the formula of calculating moving average is closer to the mental way of price weighting for investors. It is argued that the moving average *E*, out of these three averages (*E, S, W*), most accurately reflects the way investors value assets in their minds (Appel 2005).

In addition to the above-described averages, other averages are used on financial markets: moving average with free float, sinusoidally weighted moving average, moving average of one day of the week.

Moving averages are used to build transaction systems. Buy and sell signals are generated as an intersection of the moving average and the price of the financial instrument. In the literature, a lot of space is devoted to the optimization problem with the use of such systems. Meanwhile, the problem that is not well-researched is which of the applied moving averages generates better entry or exit signals and thus higher rates of return for the same analysed asset.

Therefore, the purpose of the paper is to determine the effectiveness of transaction systems based on different types of moving averages for companies listed on the Warsaw Stock Exchange, included in the three stock indexes: WIG20 (blue chips), mWIG40 (middle capitalization) and sWIG80 (small capitalization). All companies components of the WIG20, mWIG40 and sWIG80 indexes will be called in the paper WIG140 index. In the case of two types of moving averages, in which prices are weighted with the use of method 1 and method 2, respectively, it can be proved that for each of the analysed groups of companies (components of four stock indexes), a transaction system based on the first moving average type and price crossover, in most cases generates higher rates of return than the transaction system based on the second moving average type and price crossover. Similar relationships can be given for all 9 price weighting methods. By creating a ranking of moving average types (simple or weighted – see the second part of the article), one can classify all 9 types of analysed moving averages. This ranking of moving average types may be considered as an important indication for the investor of which type of moving average he should choose when building a transaction system for a company, a component of a particular stock index. It is worth emphasizing that in this approach we are interested in the method of weighing prices in moving averages. The length of the moving average is of secondary importance. And it is the length of moving averages, and not the method of weighting prices, that is the subject of other studies.

The second hypothesis of the article can be formulated as follows: transactional systems based on the moving average and price crossover differ in their stability depending on the chosen price weighting method. For some weighting methods, transaction systems will be more stable than for other weighting methods. Therefore, it is possible to construct a ranking of transaction system stability depending on the implemented price weighting method.

2 Literature review

Gartley (1935) introduced the moving average trading rule to detect stochastic trends in the prices of risky assets. According to the rule, unnecessary price fluctuations are supposedly reduced when the moving average is calculated over the price history. Black (1986) redefined the idea of Gartley (1935) by assuming that all unnecessary price fluctuations that are independent of fundamental information concerning the risky assets were noise fluctuations. Henriksson and Merton (1981) stated that trading rules are usually to determine when to buy or sell stock, and call this 'market timing'.

Pring (2014, pp. 209–232) established a division of averages due to the investment horizon. Thus, the average lengths: 10, 15, 20, 25 and 30 sessions were considered short-term, with lengths of 50, 65, 100 and 200 sessions for medium-term, and 45 week, 12, 18 and 25 month sessions as long-term. Gatley (1998) submitted another classification. According to him, short-term averages are 20 session, medium--term: 50 session and long-term: 200 session. Merrill (1992) proposed a division into moving averages of two investment horizons: 4 weeks and 26 weeks. According to Achelis (2001, pp. 203–214) the 200 session average is classified as a long-term average. In many cases, the numbers belonging to the Fibonacci sequence (e.g. 13, 21, 34, 55, 89, etc. ...) or their combinations are also used in transaction systems. In connection with the above, it can be noticed that there is no uniform division into short-, medium- and long-term moving averages which would be widely accepted on the financial markets or in the literature regarding technical analysis. The Polish stock market has adopted the use of moving averages with the following lengths: 10, 15 and 45 sessions.

Two important papers dedicated to testing transactional systems based on moving averages were those of Brock, Lakonishok and LeBaron (1992) and Faber (2007). In the first, the authors presented 26 decision-making rules using single or several moving averages based on data for the DJIA index in the period of 1897–1986. This work initiated the development of research regarding the implementation of moving averages on financial markets, including transaction costs and various time intervals. In 2013 Hang et al. (2013), following the paper of Brock, Lakonishok and LeBaron (1992) considered 10, 20, 50, 100, and 200 session moving averages, presenting a new anomaly and explaining it with new asset pricing models. In turn, Faber (2007) compared the rates of return obtained with the use of active investment strategies, based on moving averages (the main ones were systems using one moving average) with the rates of return generated by the "buy and hold" strategy (passive strategy). Faber (2007) proved for the S&P 500 index, that the rates of return obtained with the use of the strategy, based on buy and sell signals of 10-month moving average, were higher over 100 years and were characterized by lower volatility of those, obtained with the use of the "buy and hold strategy". The Faber (2007) methodology has been extended to international markets in research conducted by Gwilym et al. (2010), Moskowitz, Ooi and Pedersen (2012), Kilgallen (2012). Gwilym et al. (2010) extended the study of Faber (2007) by simulating trading in-sample based on momentum and moving average rules on international equity markets. The authors proved statistically significant profits for the momentum rule, but not taking into consideration the transaction costs. Additionally, the authors confirmed the empirical results by Faber (2007) and proved the superior risk-adjusted performance for the moving average rule when compared to the "buy and hold" strategy. Moskowitz, Ooi and Pedersen (2012), on the basis of the analysis of time-series momentum effect of prices of 58 future contracts, in the period of 1985–2009, for major asset classes (equities, bonds, currency and commodity), proved that the last 12 month returns were positive predictors for futures returns. Kilgallen (2012) repeated the study of Faber (2007) but instead of focusing on broad asset class indexes, examined returns on individual assets. The author proved that the volatility of the simple moving average strategy for individual currencies, equity indexes and commodities was on average 27% lower than the passive benchmark.

Fifield, Power and Knipe (2008) analysing investment strategies based on moving averages for 3 developed markets and 15 emerging markets in the period 1989–2003, concluded that they are more effective in emerging markets. What is even more interesting, the use of longer moving averages on the emerging markets led to higher rates of return.

Ilomaki et al. (2018) used the Dow Jones average stocks from the beginning of 1988 through to the end of 2017, and found that the lower was the moving average length, the high average daily returns, even though average volatility remained unchanged. Neely at al. (2014) on the monthly data basis found that strategies based on the moving average and price crossover are beneficial for investors. Marshall, Nguyen and Visaltanochoti (2017) draws the same conclusion in the group of US small capitalization companies.

Bolton and von Boetticher (2015) proved that in the period from 1 March 2009 to 4 April 2014 the rate of returns generated by a transaction system based on the exponential moving average were higher than the returns obtained by replicating the Johannesburg stock index ALSI TOP 40 (passive strategy). The transaction system based on SMA and EMA moving averages for stock indices in India, i.e. Nifty and Junior Nifty in the period of December 2000-November 2010 was the subject of research by Mitra (2011), who showed that only in a few cases did the transaction system based on the intersection by the moving average by price bring higher rates of return than the "buy and hold" strategy.

Hochheimer (1978) showed that the results observed for the transaction system based on the simple moving average (SMA) were higher than in the case of the exponentially weighted moving average (EMA). On the other hand, in Appel's opinion (2005), better results are generated by transaction systems using the exponential moving average (EMA) in relation to those based on the simple moving average (SMA).

Papers regarding application of the moving averages in technical analysis were also conducted on the Polish capital market. Filar and Kąkol (2013) created a transaction system based on the 12- and 100-session moving averages for seven companies included in the WIG20 stock index: Bogdanka, KGHM, PGE, PKO BP, Lotos, TVN. The analysis was conducted in the period from 1 January 2012 to 31 December 2012. The authors used three types of moving averages: simple, exponentially and linearly weighted. The results proved a higher effectiveness of medium-term than long-term moving averages. In the case of the latter, the low rates of return were caused by large fluctuations in buy and sell signals in relation to local price extremes. These fluctuations led to significant drops in the portfolio value. Juszczuk and Kozak (2016) focused on the currency market, where the relation between moving average length and the obtained results was analysed. Salamaga (2013, pp. 298–307) took into account the impact of transaction costs and influence of institutional investors on the observed returns. The work of Czuba and Kaszuba (2009) was dedicated mainly to the analysis of differences in the occurrence of buy and sell signals between various types of averages (simple, exponentially weighted and linearly weighted).

Górska (2008) created a transaction system based on the moving averages as well as on the technical oscillators. The three types of moving averages were implemented: simple, exponentially weighted and linearly), and their length was equal to 5 or 10 sessions. The author also used three technical analysis oscillators (their lengths are given in brackets): MACD (8, 17, 9 and 12, 26, 9 sessions), Momentum (5, 9 and 10, 9 sessions) and the Commodity Channel Index (5 and 10 sessions). The research allowed to draw the conclusion that trading systems using technical analysis indicators and oscillators proved to be more effective on the market of selected commodities than the system based on moving averages. In turn, in a study regarding the use of a system based on moving averages (a simple 10-session, a simple 10-session with a filter of 2.5% or a 3-day filter, two moving averages of 5- and 20-session, linear weighted average 10-session, exponential moving average 10-session), MACD oscillator (with average lengths 12, 26 and 9 sessions) and Momentum (10-session) and CCI (5-session) oscillators, tested for selected companies and indices from the Warsaw Stock Exchange and some commodities on the basis of prices from 2 March 2009 to 29 July 2011, the moving average EMA proved to be the most effective for Synthos and the WIG-Chemia index (WIG-Chemistry Index). On the other hand, the highest profit was generated by SMA in the case of the WIG-Spożywczy index (WIG-Comestible index).

Letkowski (2014) analysed the effectiveness of the transaction system based on the simple moving average of 10, 20, 30, 40, 50, 60, 70, 80, 90 and 100 sessions for selected companies included in the WIG20 index and for the WIG20 index alone, in the period of 2003–2012. The author concluded that in the case of a certain group of companies (BRE Bank, Bank Handlowy, PKN Orlen, KGHM and Bank Pekao), the best results were generated by the system using the 10 session moving average. However, for the other group of companies (Telekomunikacja Polska (today Orange), Boryszew and Asseco), the average of 100 sessions proved to be optimal.

3 Methodology

In the survey 140 companies listed on the Warsaw Stock Exchange were taken into account, whose first date of listing was earlier than 1 January 2018. The analysed companies were components (on the date 30 June 2019) of the following WSE Indexes: WIG20 (blue chips), mWIG40 (middle capitalization) and sWIG80 (small capitalization). All analysed companies were named by convention as WIG140 (WIG140 is an index defined for the purposes of the article, because WSE does not calculate such an index).

The transaction system is based on the intersection of the moving average by the closing price. For each of the analysed equities there were moving average lengths registered, for which the 20 highest returns were observed.

The following convention for KMA X was adopted later in the paper:

KMA – a specific type of moving average, i.e. S, W or E.

X – X^{th} highest return for the average KMA and for the selected equity.

For example, SMA I represents the simple moving average, with the highest return in the analysed period for a given equity, and EMA XVI is the exponentially weighted moving average, with the sixteenth highest return for a given equity.

In the process of testing transaction systems, the following assumptions were made:

1. The number of sessions for which the system was optimized is 5,000 sessions, counting back from 30 June 2019. If the company was not listed in such a long time interval, then the system was optimized in the shorter period: company's first quotation – 30 June 2019.

2. All transactions are concluded at closing prices (C – close) of the session where the buy or sell signal is generated. This approach can be considered real, taking into account the fact that after setting the closing price at the afternoon fixing (i.e. between 16:50 and 17:00), it is possible to proceed a transaction during the period of 10 minutes just at the closing price.

3. For each equity, the system was optimized for 9 different types of moving averages (*E, S, T, Tri, Var, Vol, W, Hull* and *FRAMA*). The length of the analysed averages ranged from 3 to 250 sessions, with a modification of one session.

4. Transaction costs are not included.

5. The prices of securities were taken from the bossa.pl website and adjusted for dividends paid based on information from the Notoria.pl website.

In the process of evaluating the effectiveness of transaction systems, one of the factors is its stability assessment. Three measures were calculated to assess the stability of the transaction system:

$$MM1_{KMA} = \frac{r_{KMA\ I} - r_{KMA\ XX}}{r_{KMA\ I}}$$

$$MM2 = \frac{MM1_{KMA}}{\sum_{KMA} MM1_{KMA}}$$

$$MM3 = \frac{1}{r_{KMA\ I}} \sum_{i=2}^{i=20} \left(r_{KMA\ I} - r_{KMA\ i} \right)$$

where:

$r_{KMA\ I}$ – rate of return for the moving average of $KMA\ I$,
$r_{KMA\ XX}$ – rate of return for the moving average of $KMA\ XX$,
$MM1_{KMA}$ – measure value MM1 for the moving average KMA.

The first measure is the ratio of the difference between the highest and the twentieth highest rate of return to the highest rate of return. Dividing MM1 by 19, we can obtain the average decrease of the rate of return between the highest and the twentieth highest rate of return. Thus, MM1 is used to achieve a simple ranking of moving averages. In turn, with the MM2 measure, it is possible to obtain a ranking where the weight is the rate of return attributable to each moving average. The last measure MM3 represents the sum of the differences between the highest rate of return and following it nineteen rates of return in relation to the highest one. This measure can be considered as the equivalent of distance metrics in topology.

The purpose of these three measures is to assess the stability of the transaction system. Stable transaction systems should be characterized by low values of parameters MM1, MM2 and MM3. On the other hand, large values of these measures will indicate a high variability of the rates of return obtained for different lengths of moving averages.

4 Results and discussion

4.1 Long-term investment: shares vs. stock market index

The rates of return for a simple investment strategy (buy and hold) were compared in a period in which the transaction system for each company was optimized:

$$r_i - r_{Index}$$

where:

r_i – rate of return of the share i in the analysed period,
r_{Index} – rate of return of the index, in which the company i is included, in the analysed period.

In the case of 10 companies (50%) included in the WIG20 index, the rates of return on investment in shares of these companies were higher than the rate of return of the WIG20 index. For companies included in the mWIG40 and sWIG80 indexes, these values were equal to 21 (52.50%) and 35 (43.75%) respectively. For all analysed companies (WIG140), this figure amounts to 66 companies (47.16%).

4.2 Long-term investment: transaction system vs. stock market index

For each of the analysed companies the highest rates of return obtained with the use of a transaction system, based on a moving average and price crossover, were compared with rates of return for the "buy and hold" strategy:

$$r_{KMA\ I} = r_i$$

The results are presented in Table 1.

In the case of all groups of components of the indexes: WIG20, mWIG40, sWIG80 and WIG140, the highest percentage was recorded for *T* moving average (65.00%, 52.50%, 58.75% and 57.86%, respectively). The second highest percentage in the group of components of the WIG20 index was registered for the following averages: *Tri, S* and *Vol* (50%), while for the mWIG40 index companies it was *Tri* moving average (50%), and for the sWIG80 index components, the averages *E, FRAMA, Hull, S* and *Tri* (57.50%). For all analysed companies, e.g. WIG140 index components, the second best performing average was *Tri* moving average (54.29%).

Figure 1 presents the percentage of moving averages, for which the rate of return was higher than the rate of return achieved using the "buy and hold" strategy. The distribution presented in this figure is clearly dichotomous. So it may be concluded that there are two groups of companies:

a) the first, for which the rates of return obtained with the use of the "buy and hold" strategy are always higher than those generated by the transaction system based on the intersection of prices and their moving average (mark 9 on the horizon axis);

b) the second, for which the rates of return obtained with the use of the "buy and hold" strategy are always lower than those generated by the transaction system based on the intersection of prices and their moving average (mark 0 on the horizon axis).

5 Characteristics of moving averages that optimize the transaction system

The frequency analysis was proceeded in two manners:

a) for moving averages optimizing the transaction system, i.e. KMA I,

b) for all twenty averages that yielded the highest rates of return: KMA I, KMA II, ..., KMA XX.

The prevalence of moving averages of KMA I for companies included in individual stock exchange indices is presented in Figures 2–5. All of these charts are dominated by short-term moving averages, with Hulls predominating in the case of companies included in the WIG20 index. For components of all analysed indices (WIG20, mWIG40, sWIG80 and WIG140), the frequency concentration for medium-term moving averages (> 250 sessions) is noticeable.

In addition, in Figure 2 (WIG20 components), the higher frequency of *E* moving averages lengths was observed between 64–79 session, and in Figure 3 (mWIG40 components) higher frequency was registered for *E* moving average lengths from 89 to 98 and from 104 to 116 sessions.

In the case of moving averages: KMA I, KMA II, ..., KMA XX, conclusions regarding the frequency of individual lengths of moving averages are convergent with point a), except for the WIG20 index. In the case of this index components, visible peaks appear for certain values – Table 2. For the WIG20 index components index, mainly medium to long-term moving averages prevail, except for averages: *FRAMA*, *Hull* and *T*.

The distributions of the moving averages lengths for components of the sWIG80 index are presented in the drawings 6–13. They were omitted for the remaining indexes.

6 Parity and oddness of moving average lengths

The parity and oddness of the length of moving averages (KMA I, KMA II, ..., KMA XX) were analysed. The obtained results are presented in Table 3. No apparent domination of any of the types of moving averages were observed. The obtained result is in line with expectations.

7 Analysis of the effectiveness of transaction systems based on individual moving averages

Tables 4–7 present the percentage of cases when $r_{KMA_1 I} \geq r_{KMA_2 I}$, under the condition $(KMA_1 I \neq KMA_2 I)$ e.g. for different types of moving averages, for companies included in analysed stock exchange indices.

For any pair of moving averages KMA 1 I and KMA 2 I, it was not observed that the percentage of cases when $r_{KMA_1 I} \geq r_{KMA_2 I}$, was equal to 100%. The maximum values of this percentage for the components of analysed indices was registered for the following moving average pairs (frequencies are indicated in brackets):

WIG20: T ≥ Hull (95%), T ≥ Vol, S ≥ FRAMA and Tri ≥ FRAMA (90%),

mWIG40: T ≥ Hull (95%) and Tri ≥ Var (85%),

sWIG80: Tri ≥ Var (90%) and T ≥ Hull (77.50%),

WIG_{140}: T ≥ Hull (85%), E ≥ Tri and Tri ≥ Var (85%).

Tables 4–7 presented in the form of three-dimensional graphs (Figures 14–17) allow to draw the conclusion that the shapes (not values) of charts are similar for all indices, with the exception of the WIG20 index.

8 Ranking of transaction systems effectiveness

The effectiveness of transactional systems and their assessment was described, among others, in the works of LeBeau and Lucas (1991), Katsanos (2009, pp. 149–165), Kaufman (2013, pp. 58–59) and Zalewski (2001). For the purposes of this article, the ranking of transaction systems effectiveness was prepared in two ways:

1. Ranking R1. For each security, the moving average lengths of KMA I were obtained, which optimized the transaction system by giving rates of return: $r_{KMA\ I}$. The highest rate of return, the highest position in the ranking of the analysed moving average: the highest rate of return is position 1, the second in order of the rate of return – second position in the ranking (2), etc. After assigning a moving average ranking according to the rate of return obtained for the analysed securities, the rankings for each moving average type were aggregated. This moving average, which obtained the lowest (highest) sum, was categorized at the first (last) place.

2. Ranking R2. Due to the fact that the ranking prepared in the first point does not take into account differences in rates of return for individual moving averages for the same security, it was necessary to create a ranking without such a defect. For a given security, the moving average KMA has been assigned a weight:

$$w_{KMA_i} = \frac{r_{KMA\,I\,i}}{\sum_{i=1}^{i=9} r_{KMA\,I_i}}$$

Analogically, weights were assigned to the remaining moving averages. In the next step, w_{KMA} weights were aggregated for all analysed securities, and then a ranking of analysed averages was made according to the rule that the highest value in the ranking contributed to the average with the highest value of the sum of weights.

In the ranking R1, the highest rates of return were obtained by moving average *T*, and moving average *Tri* was in second place. This applies to components of all analysed indices (WIG20, mWIG40, sWIG80 and WIG140). The third place was taken by the average *S*, which for the mWIG40 index companies was in the fifth place. In the case of the fourth place, it is worth mentioning the average *Vol* (for companies from the mWIG40 and sWIG80 indexes) or *W* (for companies from the WIG20 and WIG140 indexes). The last places in the ranking include such moving averages as *Var, Hull, FRAMA* and *E*.

Taking into account the sum of points in the ranking, which each moving average has obtained, the final ranking would be presented as follows (in brackets the number of points): *T* (4), *Tri* (8), *S* (14), *W* (16), *Vol* (19), *E* (27), *FRAMA* (29), *Hull* (30), *VAR* (33).

In the R2 ranking, as in the R1 ranking, the first two places belong to the moving averages *T* and *Tri*. The latter ranked third for companies from the sWIG80 index. Classification of the third position seems to be more difficult than in the case of the R1 ranking, as the average *S* took third place for components of WIG20 and WIG140 indices, second place for companies from the sWIG80 index and fifth for companies from the mWIG40 index. Moving average *Vol* was third for mWIG40 components, twice fourth (sWIG80 and WIG140) and seventh for WIG20 companies. Taking into account the sum of points in the ranking which each moving average obtained, the final ranking would be presented as follows (in brackets the number of points): *T* (4), *Tri* (9), *Vol* (14), *S* (15), *W* (18), *E* (24), *FRAMA* (30), *Hull* (32) and *Var* (34). In both rankings, the moving average *T* and *Tri* occupy the first two places. In turn, average moves *S, W* and *Vol* swapped in both rankings places in third, fourth and fifth place. Sixth position in the R1 and R2 ranking belongs to moving average *E*. The last three positions occupy the same moving averages in both rankings: *FRAMA, Hull* and *Var*.

9 Ranking transaction systems stability

In the case of the measure MM1, the lowest values in the ranking were (in brackets the sum of the ranking value for the components of all four indices was given) moving average *W* (8), before moving average *E* (10) and moving average *TRI* (12) – Table 10. On the other hand, moving average *T* (35) was ranked before *Hull* (33). Moving average *W* was classified in first position for all indexes except mWIG40. In turn, moving average *E* ranked second for all indices except the mWIG40 index. Moving average *T* was in last place, and *Hull* moving average was penultimate for all indexes except mWIG40.

For the MM2 measure, the following moving averages were classified in the highest positions in the ranking: *E* (8), *Tri* (10) and *W* (11), while in the worst: *T* (36) and *Hull* (31) – Table 11. In this ranking there is no similar regularity as was observed in the case of the ranking for the MM1 measure. This applies to the top ranking positions. The moving average *E* was classified twice in the first (sWIG80 and WIG140) and the third position (WIG20 and mWIG40). In turn, the average *TRI* – twice in the second (sWIG80 and WIG140), once in the first (mWIG40) and once in the fifth position (mWIG40). The compatibility of the indications for the last places is much better. The moving average *T* was placed in the last position for all indices, the moving average *Hull* – three times in eighth and seventh (WIG20), and the moving average *FRAMA* – three times in seventh and one in eighth position (WIG20).

For the MM3 measure, the best results were observed for the following averages: *T* (4), *Hull* (10) and *FRAMA* (14), while the worst were observed for *W* and *E* (in both cases 33) and *Tri* (27) – Table 12. In this ranking the moving average *T* was ranked first for all four indices. *Hull* moving average was twice classified in second place (mWIG40 and WIG140) and twice in third position (WIG20 and sWIG80). Two moving averages, which were classified in the last position, i.e. *W* and *E*, obtained the following positions in the ranking. The first moving average placed twice in ninth place (WIG20 and WIG140), in seventh (mWIG40) and eighth (sWIG80) positions. The second moving average – three times in eighth position (WIG20, mWIG40 and WIG140) and once in ninth place (sWIG80).

10 Conclusions

The article presents an analysis of the effectiveness of investments according to the indications of nine different types of moving averages. It was the aim of this paper to treat the indications in a collective manner rather than individually for each security. The literature review clearly indicates that this last approach dominates. Therefore, this paper tries to fill the research gap and find general conclusions specific to certain groups of assets, and not just to one class only.

The most important conclusions that can be drawn from the study are as follows:

1. For the analysed companies, the percentage when the highest rate of return obtained with the use of a transaction system, based on a moving average and price crossover, was higher than the rate of return for the "buy and hold" strategy, reached its maximum for the moving average *T*. The highest percentage was observed for the components of the following indexes: WIG20 (65.00%), sWIG80 (58.75%), WIG140 (57.86%) and mWIG40 (52.50%). This conclusion is partly consistent with the results presented in the works of Mitra (2011), Bolton and von Boetticher (2015), Neely et al. (2014) and Marshall, Nguyen and Visaltanochoti (2017). In the case of the work of Mitra (2011) as well as of Bolton and von Boetticher (2015), most of the rates of return obtained through the transaction system were

higher than in the "buy and hold" strategy. However, Neely et al. (2014) as well as Marshall, Nguyen and Visaltanochoti (2017) focused on monthly rather than daily rates of return.

2. On the Polish capital market, the highest returns were achieved by short-term moving averages, whose length is equal to a few sessions. This fact allows to draw the conclusion that speculative transactions prevail on this market. This observation confirms the conclusion obtained by Ilomaki, Laurila and McAleer (2018) on the American financial market and the results of Górska (2011) and partially those of Filar and Kąkol (2013) and of Letkowski (2014) on the Polish market.

3. The analysis of parity and imparity of moving average lengths optimizing the trading system revealed that none of them, i.e. even or odd, were dominant on the Polish market. This result confirms previous expectations based on statistics. It would be surprising if one of the moving averages, i.e. even or odd, occurred more often than the other.

4. In the case of any of the moving averages, there was no coincidence that, however, the one type of moving average was always (100%) generating higher rates of return than the other type for all the analysed companies. This conclusion is in line with the paper of Górska (2011). For some of the analysed moving average pairs, however, this percentage was very high, such as for the pair of moving averages: *T* and *Hull* (WIG20, mWIG40) or *Tri* and *Var* (sWIG80). For the components of all the analysed indices, the transaction system based on the moving average *E* proved to be less effective than that based on the average *S*. This conclusion confirms the results of Hochheimer (1978) and denies the opinion of Appel (2005).

5. In the performance ratings of the individual moving averages (R1 and R2), at the highest positions classified the averages: *T* and *Tri* and at the lowest, the following moving averages: *FRAMA*, *Hull* and *Var*.

6. The trading systems stability rankings proved that according to measures MM1 and MM2, the most stable seemed to be systems based on averages: *W*, *E* and *TRI*, while the least stable were those using moving averages: *T* and *Hull*. Therefore, systems based on a time series moving average generate the highest rates of return, but at the same time prove to be the least stable. In the case of the MM3 measure, the conclusion was completely different. It seems that the issue of the stability of trading systems should be researched in a separate article.

The most important limitation of the study was the different lengths of investment horizons for individual companies, the components of stock indices. On the one hand, selecting a long time horizon should provide more meaningful results, but on the other hand, comparisons between the results obtained for different companies are somewhat hindered.

Likewise, the research should be continued for other companies listed on the WSE in Warsaw, other stock exchanges, as well as for asset classes such as currencies, commodities and for stock indices. Another approach would be sector selection of companies, and not according to their capitalization and turnover (in this way shares are classified into Wig20, mWIG40 and sWIG80 indices).

References

Achelis S. (2001), *Technical Analysis from A to Z*, Mc-Graw Hill.

Appel G. (2005), *Technical Analysis Power Tools for Active Investors*, Prentice Hall Publishing.

Arms R. (1987), *Volume Cycles in the Stock Market*, McGraw-Hill Education.

Black F. (1986), Noise, *Journal of Finance*, 41, 528–543.

Bolton J., von Boetticher S. (2015), Momentum trading on the Johannesburg Stock Exchange after the global financial crisis, *Procedia Economics and Finance*, 24, 83–92.

Brock W., Lakonishok H., LeBaron B. (1992), Simple technical trading rules and the stochastic properties of stock returns, *Journal of Finance*, 47, 1731–1764.

Czuba M., Kaszuba B. (2009), Porównanie efektów stosowania średnich ruchomych w analizie finansowych szeregów czasowych polskiego rynku akcji, *Prace Naukowe Uniwersytetu Ekonomicznego we Wrocławiu. Nauki o Finansach*, 75, 22–35.

Ehlers J. (2005), Fractal adaptive moving average, *Technical Analysis of Stock & Commodities*, 23(10), 81–82.

Faber M. (2007), A quantitative approach to tactical asset allocation, *Journal of Wealth Management*, 9, 69–79.

Fifield S., Power D., Knipe D. (2008), The performance of moving average rules in emerging stock markets, *Applied Financial Economics*, 18, 151–153.

Filar W., Kąkol W. (2013), Znaczenie średnich ruchomym w podejmowaniu decyzji inwestycyjnych na giełdzie, *Modern Management Review*, 18(20), 31–41.

Gardner M. (2010), Trading indexes with the Hull moving averages, *Technical Analysis of Stock & Commodities*, 28(12), 19–23.

Gatley E. (1998), *Forecasting Profits Using Price and Time*, Wiley & Sons.

Gartley H. (1935), *Profits in the Stock Markets*, Lambert-Gann Publishing.

Górska A. (2008), Zastosowanie narzędzi analizy technicznej w bezpośrednim i pośrednim inwestowaniu w towary, *Zeszyty Naukowe Szkoły Głównej Gospodarstwa Wiejskiego. Ekonomika i Organizacja Gospodarki Żywności*, 71, 53–65.

Górska A. (2011), Wykorzystanie strategii inwestycyjnych opartych na analizie technicznej do handlu towarami z WGT SA, *Zeszyty Naukowe Szkoły Głównej Gospodarstwa Wiejskiego. Problemy Rolnictwa Światowego*, 26(11), 67–76.

Gwilym O., Clare A., Seaton J., Thomas S. (2010), Price and momentum as robust tactical approaches to global equity investing, *Journal of Investing*, 19, 80–91.

Henriksson R., Merton R. (1981), On market timing and investment performance. An equilibrium theory of value for market forecast, *Journal of Business*, 54, 513–515.

Hochheimer C. (1978), *Computers can help you to trade the futures markets*, Commodity Yearbook, Commodity Research Bureau.

Ilomaki J., Laurila H., McAleer M. (2018), Market timing with moving averages, *Sustainability*, 10, 21–25.

Juszczuk P., Kozak J. (2016), Paradygmat programowania proceduralnego w procesie budowy systemów automatycznych bazujących na średnich kroczących, *Studia Informatica Pomerania*, 39, 25–35.

Katsanos M. (2009), *Intermarket Trading Strategies*, John Wiley & Sons.

Kaufman P. (1978), *Commodity Trading Systems and Methods*, John Wiley & Sons.

Kaufman P. (2013), *Trading Systems and Methods*, John Wiley & Sons.

Keltner C. (1960), *How to Make Money in Commodities*, The Keltener Statistical Service.

Kilgallen T. (2012), Testing the simple moving average across commodities, global stock indices, and currencies, *Journal of Wealth Management*, 15, 82–100.

LeBeau C., Lucas D. (1991), *Technical Traders Guide to Computer Analysis of the Futures Markets*, Mc-Graw Hill Education.

Letkowski D. (2014), Wykorzystanie średnich ruchomych w analizie inwestycji giełdowych – dobór modelu i długość próby, *Acta Universitatis Lodziensis, Folia Oeconomica*, 301(2), 167–177.

Marshall B., Nguyen N., Visaltanochoti N. (2017), Time series momentum and moving average trading rules, *Quantitative Finance*, 17, 405–421.

Merrill A. (1992), Moving average crossover, *Technical Analysis of Stock & Commodities*, 10(8), 332–335.

Metastock for Windows 95 & NT (2000), *User's manual*, Equis International.

Mitra S. (2011), Usefulness of moving average based trading rules in India, *International Journal of Business and Management*, 6(7), 199–206.

Moskowitz T., Ooi Y., Pedersen L. (2012), Time series momentum, *Journal of Financial Economics*, 104, 228–250.

Neely C., Rapach D., Tu J., Zhou G. (2014), Forecasting equity risk premium: the role of technical indicators, *Management Science*, 66, 1772–1791.

Payne J. (1989), A better way to smooth data, *Technical Analysis of Stock & Commodities*, 7(10), 326–333.

Pring M. (2014), *Technical Analysis Explained*, McGraw Hill.

Salamaga P. (2013), Zastosowanie metody średniej kroczącej do badania zyskowności inwestycji na polskim rynku kapitałowym, *Prace Naukowe Uniwersytetu Ekonomicznego we Wrocławiu. Nauki o Finansach*, 323, 298–307.

Sklarew A. (1980), *Technical of Professional Commodity Chart Analysis*, Commodity Research Bureau.

Thusar C. (1992), Adapting moving averages to market volatility, *Technical Analysis of Stocks & Commodities*, 10(3), 108–114.

Zalewski G. (2001), *Kontrakty terminowe w praktyce*, WIG-PRESS.

Appendix

Table 1
Percentage of companies for each of the averages when the rate of return obtained with the use of a trading system was higher than holding the company's shares ("buy and hold" strategy) in the analysed period

Components of	E	FRAMA	Hull	S	T	Tri	Var	Vol	W
WIG20	45.00	45.00	45.00	50.00	65.00	50.00	45.00	50.00	45.00
mWIG40	47.50	45.00	42.50	47.50	52.50	50.00	47.50	47.50	47.50
sWIG80	57.50	57.50	57.50	57.50	58.75	57.50	56.25	47.50	57.50
WIG140	52.86	52.14	51.43	53.57	57.86	54.29	52.14	53.57	52.86

Source: own calculation.

Table 2
Lengths of moving averages, near which peaks of frequency were observed

Moving average	Lengths of moving averages
E	27–46, 215–224
FRAMA	9–14, 29–31, 134–135, 199
Hull	3, 7, 23, 74, 220–245, 281, 287
S	12, 34–41
T	3–12, 51, 71, 276, 286
TRI	36, 45, 259–264, 295–300
Var	20, 70–71
Vol	50, 280–300
W	51, 267–296

Source: own calculation.

Table 3
The frequency of occurrence of even and odd lengths of moving averages (%)

Moving average lengths	E	FRAMA	Hull	S	T	Tri	Var	Vol	W
				WIG20					
Even	49.75	51.00	48.00	49.25	48.50	49.50	51.25	48.75	49.25
Odd	50.25	49.00	52.00	50.75	51.50	50.50	48.75	51.25	50.75
				mWIG40					
Even	50.25	50.63	50.50	48.63	50.50	50.63	48.88	49.00	50.75
Odd	49.75	49.38	49.50	51.38	49.50	49.38	51.13	51.00	49.25
				sWIG80					
Even	49.81	49.56	50.69	49.31	49.88	49.75	50.19	49.56	50.19
Odd	50.19	50.44	49.31	50.69	50.13	50.25	49.81	50.44	49.81
				WIG140					
Even	49.93	50.07	50.25	49.11	49.86	49.96	49.96	49.29	50.21
Odd	50.07	49.93	49.75	50.89	50.14	50.04	50.04	50.71	49.79

Source: own calculation.

Table 4
Percentage of cases when $r_{KMA_1 I} \geq r_{KMA_2 I}$ for WIG20

	E	**FRAMA**	**Hull**	**S**	**T**	**Tri**	**Var**	**Vol**
E ≥		65.00	40.00	45.00	15.00	75.00	75.00	50.00
FRAMA ≥	*35.00*		40.00	10.00	15.00	10.00	25.00	20.00
Hull ≤	*60.00*	*60.00*		45.00	5.00	45.00	45.00	50.00
S ≥	*55.00*	*90.00*	*55.00*		15.00	20.00	70.00	65.00
T ≤	*85.00*	*85.00*	*95.00*	*85.00*		80.00	85.00	90.00
Tri ≥	*25.00*	*90.00*	*55.00*	*80.00*	*20.00*		75.00	75.00
Var ≥	*25.00*	*75.00*	*55.00*	*30.00*	*15.00*	*25.00*		45.00
Vol ≥	*50.00*	*80.00*	*50.00*	*35.00*	*10.00*	*25.00*	*55.00*	
W ≥	*60.00*	85.00	*55.00*	*35.00*	*15.00*	*25.00*	*60.00*	*60.00*

Source: own calculation.

Table 5
Percentage of cases when $r_{KMA_1 I} \geq r_{KMA_2 I}$ for mWIG40

	E	**FRAMA**	**Hull**	**S**	**T**	**Tri**	**Var**	**Vol**	**W**
E ≥		70.00	60.00	27.50	32.50	57.50	85.00	27.50	47.50
FRAMA ≥	*30.00*		60.00	32.50	30.00	27.50	52.50	35.00	27.50
Hull ≥	*40.00*	*40.00*		27.50	5.00	25.00	40.00	17.50	25.00
S ≥	*72.50*	*67.50*	*72.50*		32.50	30.00	77.50	55.00	40.00
T ≥	*67.50*	*70.00*	*95.00*	*67.50*		57.50	82.50	65.00	70.00
Tri ≥	*42.50*	*72.50*	*75.00*	*70.00*	*42.50*		85.00	47.50	70.00
Var ≥	*15.00*	*47.50*	*60.00*	*22.50*	*17.50*	*15.00*		27.50	25.00
Vol ≥	*72.50*	*65.00*	*82.50*	*45.00*	*35.00*	*52.50*	*72.50*		47.50
W ≥	*52.50*	*72.50*	*75.00*	*60.00*	*30.00*	*30.00*	*75.00*	*52.50*	

Source: own calculation.

Table 6
Percentage of cases when $r_{KMA_1 I} \geq r_{KMA_2 I}$ for sWIG80

		FRAMA	**Hull**	**S**	**T**	**Tri**	**Var**	**Vol**	**W**
E ≥			60.00	33.75	48.75	56.25	80.00	26.25	50.00
FRAMA ≥	*55.00*		60.00	36.25	32.50	38.75	61.25	37.50	38.75
Hull ≥	*40.00*	*40.00*		33.75	22.50	32.50	48.75	35.00	36.25
S ≥	*66.25*	*63.75*	*66.25*		48.75	46.25	73.75	53.75	66.25
T ≥	*51.25*	*67.50*	*77.50*	*51.25*		56.25	75.00	57.50	62.50
Tri ≥	*43.75*	*61.25*	*67.50*	*53.75*	*43.75*		80.00	56.25	72.50
Var ≥	*20.00*	*38.75*	*51.25*	*26.25*	*25.00*	*20.00*		26.25	30.00
Vol ≥	*73.75*	*62.50*	*65.00*	*46.25*	*42.50*	*43.75*	*73.75*		50.00
W ≥	*50.00*	*61.25*	*63.75*	*33.75*	*37.50*	*27.50*	*70.00*	*50.00*	

Source: own calculation.

Table 7
Percentage of cases when $r_{KMA_1 I} \geq r_{KMA_2 I}$ for WIG140

	E	**FRAMA**	**Hull**	**S**	**T**	**Tri**	**Var**	**Vol**	**W**
E ≥		55.00	57.14	33.57	39.29	59.29	80.71	30.00	47.86
FRAMA ≥	*45.00*		57.14	31.43	29.29	31.43	53.57	34.29	32.14
Hull ≥	*42.86*	*42.86*		33.57	15.00	32.14	45.71	32.14	34.29
S ≥	*66.43*	*68.57*	*66.43*		39.29	37.86	74.29	55.71	58.57
T ≥	*60.71*	*70.71*	*85.00*	*60.71*		60.00	78.57	64.29	67.86
Tri ≥	*40.71*	*68.57*	*67.86*	*62.14*	*40.00*		80.71	56.43	72.14
Var ≥	*19.29*	*46.43*	*54.29*	*25.71*	*21.43*	*19.29*		29.29	30.00
Vol ≥	*70.00*	*65.71*	*67.86*	*44.29*	*35.71*	*43.57*	*70.71*		47.86
W ≥	*52.14*	*67.86*	*65.71*	*41.43*	*32.14*	*27.86*	*70.00*	*52.14*	

Table 8
Ranking R1

Component of the index		E	FRAMA	Hull	S	T	Tri	Var	Vol	W
WIG20	Points	125	146	111	86	41	68	113	112	98
	Place in ranking	**8**	**9**	**5**	**3**	**1**	**2**	**7**	**6**	**4**
mWIG40	Points	201	241	275	181	124	140	259	172	165
	Place in ranking	**6**	**7**	**9**	**5**	**1**	**2**	**8**	**4**	**3**
sWIG80	Points	453	430	488	318	303	306	509	360	378
	Place in ranking	**7**	**6**	**8**	**3**	**1**	**2**	**9**	**4**	**5**
WIG140	Points	779	817	874	585	468	514	881	644	641
	Place in ranking	**6**	**7**	**8**	**3**	**1**	**2**	**9**	**5**	**4**
Sum of places in ranking		**27**	**29**	**30**	**14**	**4**	**8**	**33**	**19**	**16**

Source: own calculation.

Table 9
Ranking R2

		E	FRAMA	Hull	S	T	Tri	Var	Vol	W
WIG20	Points	195.68	177.20	212.96	224.93	327.67	234.28	210.76	206.34	210.19
	Place in ranking	**6**	**7**	**9**	**5**	**1**	**2**	**8**	**3**	**4**
mWIG40	Points	443.69	401.59	394.76	460.11	504.18	467.15	401.58	466.45	460.49
	Place in ranking	**6**	**7**	**9**	**5**	**1**	**2**	**8**	**3**	**4**
sWIG80	Points	858.94	851.66	852.17	931.26	961.85	928.26	807.39	912.21	896.26
	Place in ranking	**6**	**8**	**7**	**2**	**1**	**3**	**9**	**4**	**5**
WIG140	Points	1498.31	1430.45	1459.89	1616.30	1793.70	1629.68	1419.73	1584.99	1566.94
	Place in ranking	**6**	**8**	**7**	**3**	**1**	**2**	**9**	**4**	**5**
Sum of places in ranking		**24**	**30**	**32**	**15**	**4**	**9**	**34**	**14**	**18**

Source: own calculation.

Table 10
Ranking of stability of transaction system – measure MM1

Index	E	FRAMA	Hull	S	T	Tri	Var	Vol	W
Wig20	81 **2**	114 **7**	116 **8**	85 **3**	144 **9**	89 **4**	101 **6**	96 **5**	74 **1**
mWIG40	168 **4**	211 **7**	265 **9**	165 **2**	264 **8**	157 **1**	167 **3**	189 **6**	174 **5**
sWIG80	368 **2**	386 **5**	443 **8**	398 **6**	461 **9**	378 **4**	401 **7**	371 **3**	353 **1**
WIG140	617 **2**	711 **7**	824 **8**	648 **4**	869 **9**	624 **3**	669 **6**	656 **5**	601 **1**
Sum of rankings for moving averages	**10**	**26**	**33**	**15**	**35**	**12**	**22**	**19**	**8**

Source: own calculation.

Table 11
Ranking of stability of transaction system – measure MM2

Index	E	FRAMA	Hull	S	T	Tri	Var	Vol	W
WIG20	164.07 **3**	274.87 **8**	264.27 **7**	159.47 **2**	414.64 **9**	169.25 **5**	234.51 **6**	168.91 **4**	150.00 **1**
mWIG40	310.01 **3**	493.14 **7**	669.80 **8**	305.43 **2**	721.78 **9**	303.95 **1**	388.47 **6**	381.31 **5**	326.10 **4**
sWIG80	744.42 **1**	942.03 **7**	1047.94 **8**	819.68 **5**	1083.76 **9**	779.69 **2**	886.75 **6**	811.42 **4**	784.31 **3**
WIG140	1218.50 **1**	1710.04 **7**	1982.01 **8**	1284.59 **4**	2220.18 **9**	1252.90 **2**	1509.74 **6**	1361.65 **5**	1260.40 **3**
Sum of rankings for moving averages	**8**	**29**	**31**	**13**	**36**	**10**	**24**	**18**	**11**

Source: own calculation.

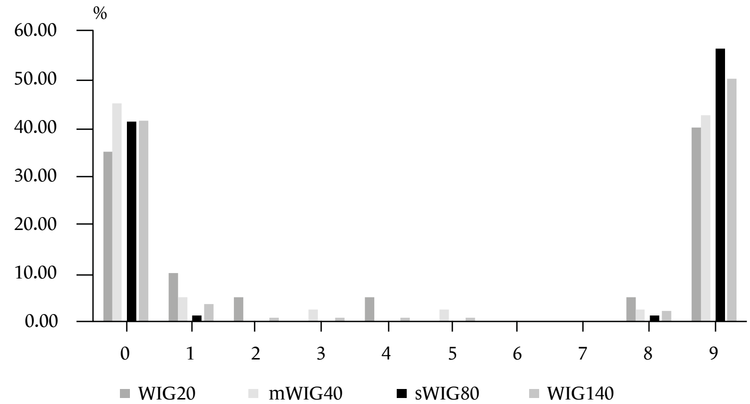
Table 12

Ranking of stability of transaction system – measure MM3

Index	**E**	**FRAMA**	**Hull**	**S**	**T**	**Tri**	**Var**	**Vol**	**W**
WIG20	118	80	81	109	57	114	101	108	132
	8	**2**	**3**	**6**	**1**	**7**	**4**	**5**	**9**
mWIG40	223	194	130	214	126	228	215	209	221
	8	**3**	**2**	**5**	**1**	**9**	**6**	**4**	**7**
sWIG80	434	406	373	364	347	396	386	428	429
	9	**6**	**3**	**2**	**1**	**5**	**4**	**7**	**8**
WIG140	775	680	584	687	530	738	702	745	782
	8	**3**	**2**	**4**	**1**	**6**	**5**	**7**	**9**
Sum of rankings for moving averages	**33**	**14**	**10**	**17**	**4**	**27**	**19**	**23**	**33**

Source: own calculation

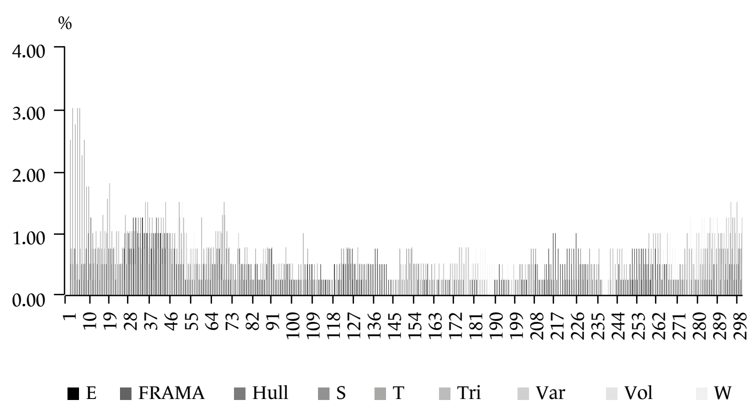
Figure 1
The percentage of moving averages, when the rate of return using the trading system was higher than the rate of return in the “buy and hold” strategy for the analysed companies



Note: the horizontal axis indicates how many times such a relationship has taken place.

Source: own calculation.

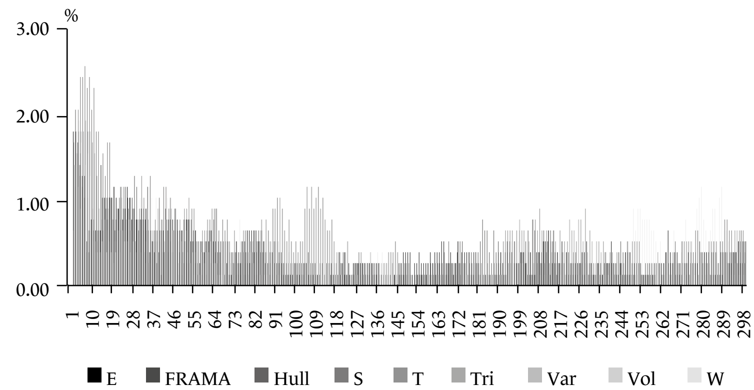
Figure 2
Frequency of moving averages lengths depending on the type of the moving average – index WIG20



Source: own calculation.

Figure 3

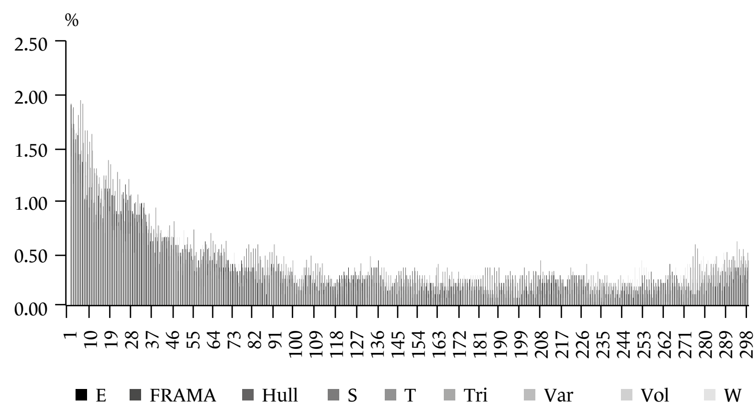
Frequency of moving averages lengths depending on the type of the moving average – index mWIG40



Source: own calculation.

Figure 4

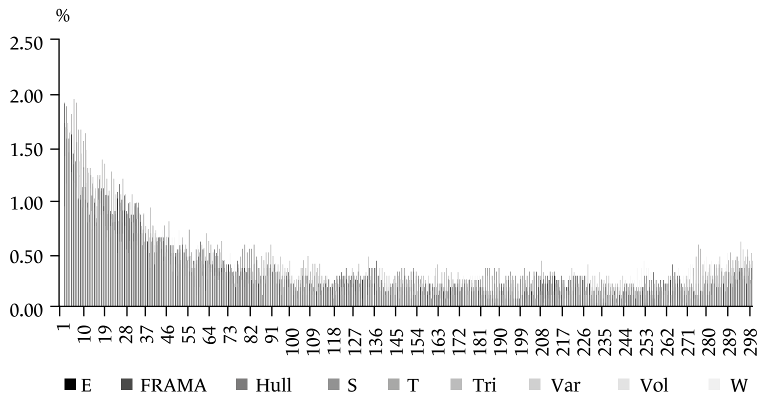
Frequency of moving averages lengths depending on the type of the moving average – index sWIG80



Source: own calculation.

Figure 5

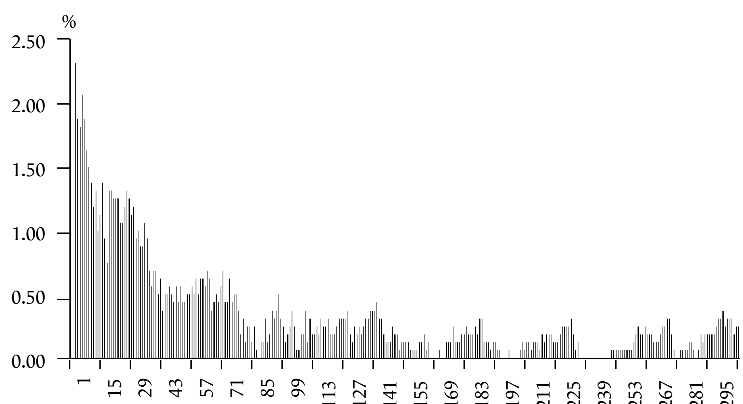
Frequency of moving averages lengths depending on the type of the moving average – index WIG140



Source: own calculation.

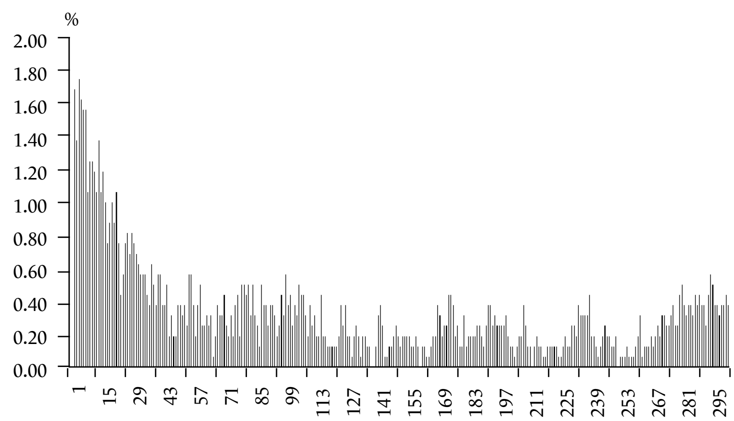
Figure 6

Moving averages length distribution for WIG80 index components, moving average type *E*



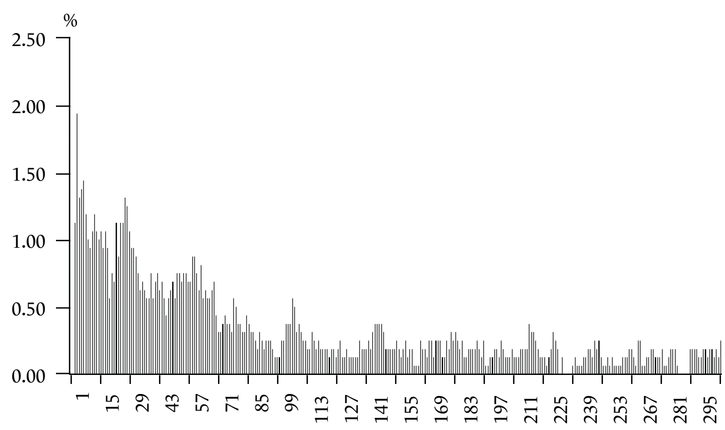
Source: own calculation.

Figure 7
Moving averages length distribution for WIG80 index components, moving average type *FRAMA*



Source: own calculation.

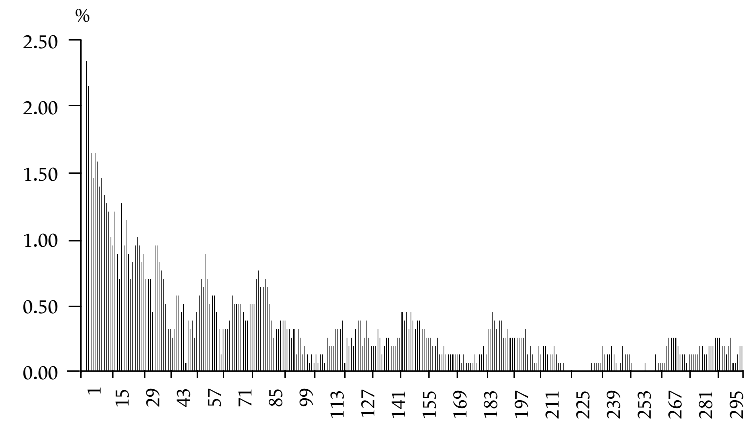
Figure 8
Moving averages length distribution for WIG80 index components, moving average type *Hull*



Source: own calculation.

Figure 9

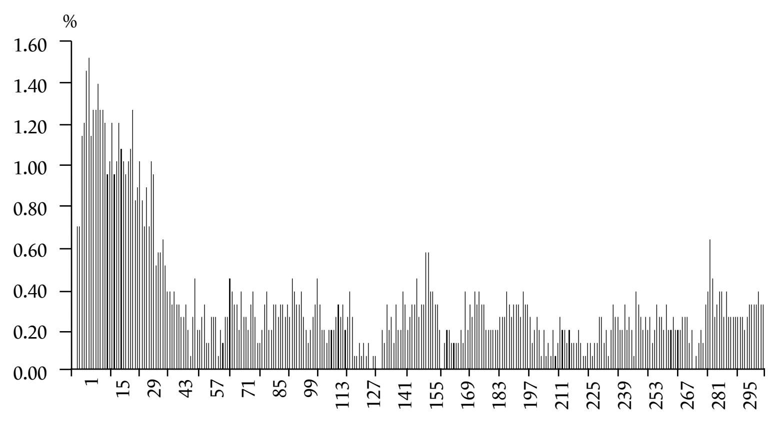
Moving averages length distribution for WIG80 index components, moving average type *S*



Source: own calculation.

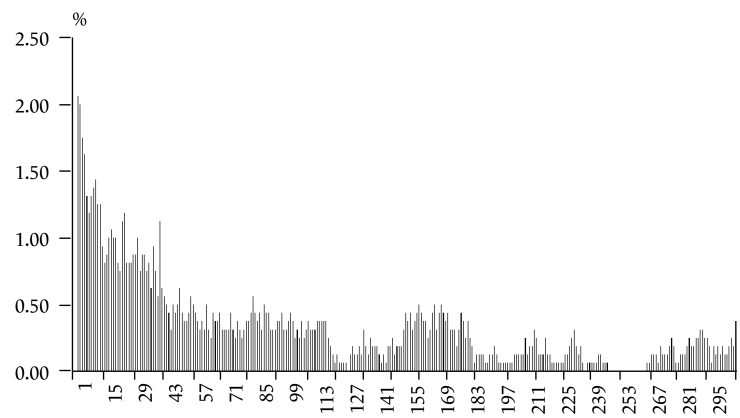
Figure 10

Moving averages length distribution for WIG80 index components, moving average type *T*



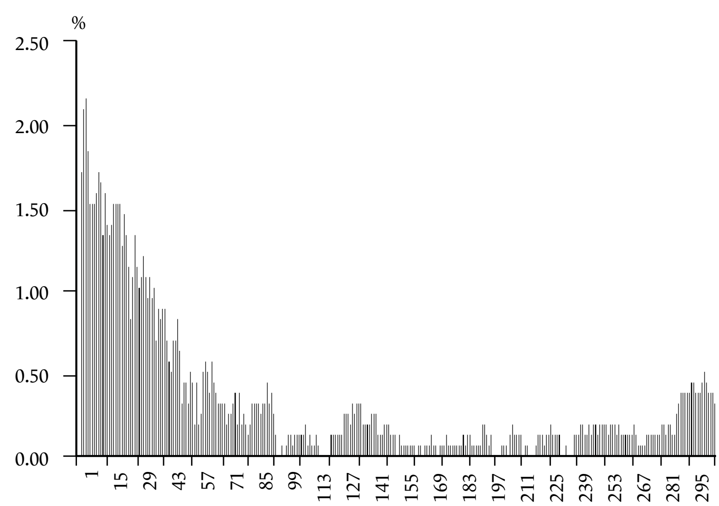
Source: own calculation.

Figure 11
Moving averages length distribution for WIG80 index components, moving average type *Tri*



Source: own calculation.

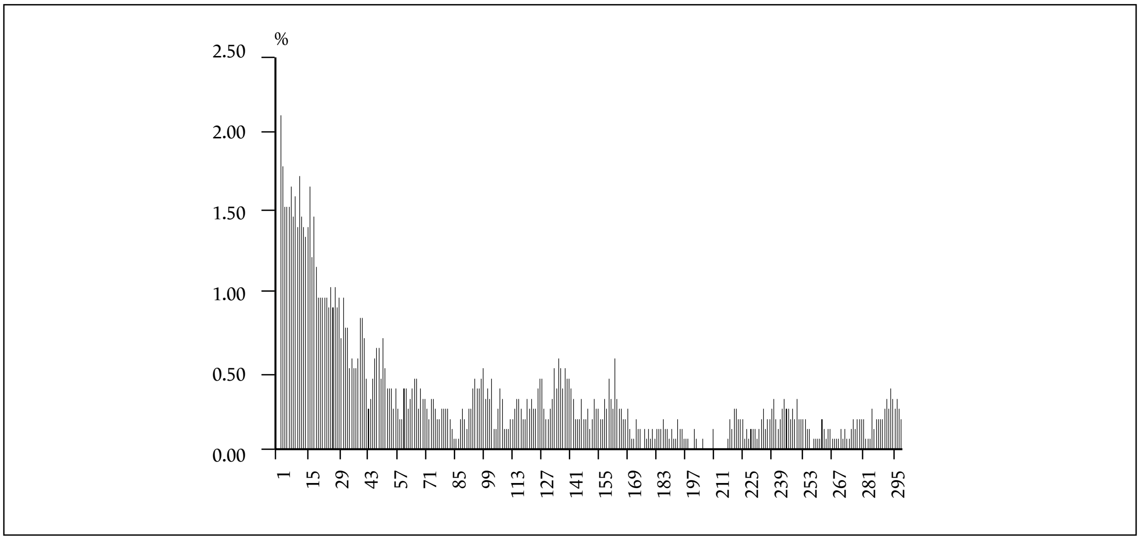
Figure 12
Moving averages length distribution for WIG80 index components, moving average type *Var*



Source: own calculation.

Figure 13

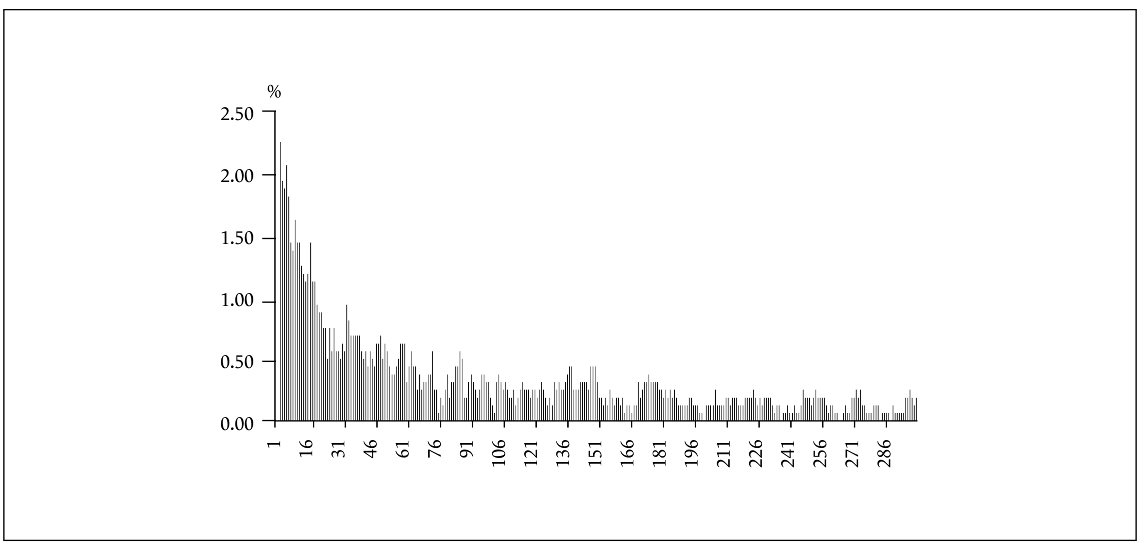
Moving averages length distribution for WIG80 index components, moving average type *Vol*



Source: own calculation.

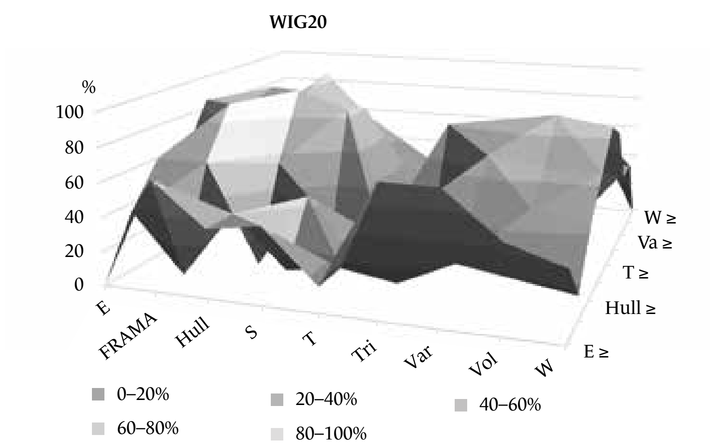
Figure 14

Moving averages length distribution for WIG80 index components, moving average type *W*



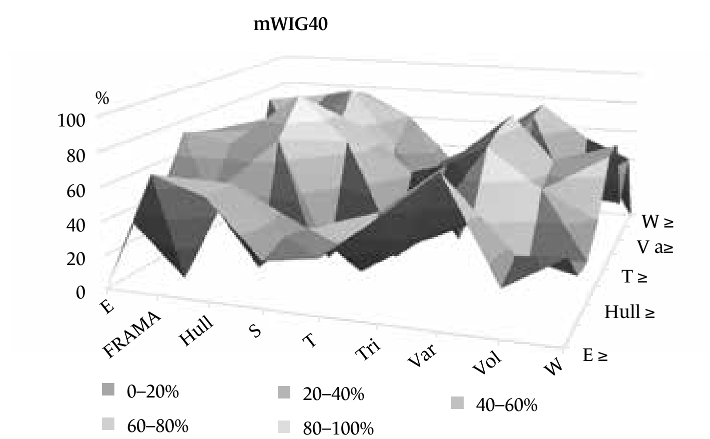
Source: own calculation.

Figure 15
Three dimensional presentation of results from Table 4



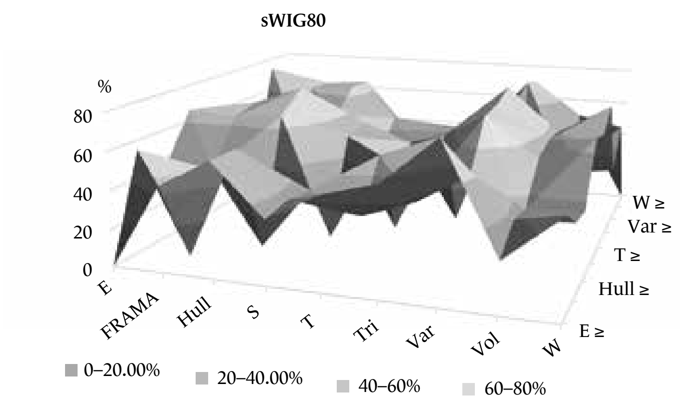
Source: own calculation.

Figure 16
Three dimensional presentation of results from Table 5



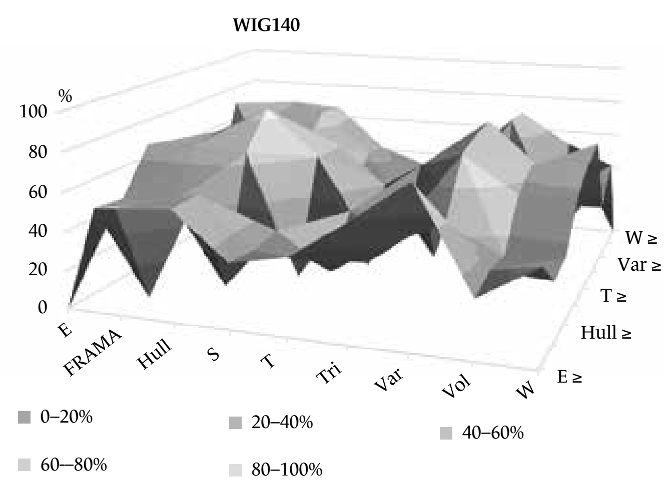
Source: own calculation.

Figure 17
Three dimensional presentation of results from Table 6



Source: own calculation.

Figure 18
Three dimensional presentation of results from Table 7



Source: own calculation.

Efektywność i stabilność systemów transakcyjnych wykorzystujących sygnały kupna i sprzedaży 9 różnych średnich ruchomych na przykładzie cen 140 akcji należących do 3 indeksów GPW w Warszawie

Większość artykułów oceniających skuteczność systemów transakcyjnych koncentruje się na optymalizacji parametrów systemu dla jednego instrumentu finansowego. Dokonany przegląd literatury wyraźnie wskazuje, że dominuje właśnie takie podejście, tj. indywidualne dla analizowanych aktywów finansowych. Widoczny jest brak badań obejmujących problem skuteczności systemów transakcyjnych wykorzystujących średnie kroczące w szerszej ramce, tj. określenie, które średnie kroczące statystycznie generują najlepsze sygnały kupna i sprzedaży. W artykule chodzi o potraktowanie płynących wskazań w sposób zbiorowy (statystyczny), a nie indywidualnie dla każdego papieru wartościowego. Tym samym artykuł stara się wypełnić powstałą lukę badawczą i sformułować wnioski ogólne, charakterystyczne dla pewnej grupy aktywów, a nie tylko dla jednego, konkretnego.

Celem artykułu jest określenie skuteczności systemów transakcyjnych na podstawie 9 różnych rodzajów średnich kroczących dla spółek notowanych na Giełdzie Papierów Wartościowych w Warszawie, należących do trzech indeksów giełdowych: WIG20 (*blue chips*), mWIG40 (średnia kapitalizacja) i sWIG80 (mała kapitalizacja), tj. dla 140 spółek ogółem. Wskazania kupna generowane były w systemie transakcyjnym, gdy cena zamknięcia przebijała od dołu określoną średnią ruchomą, a sygnały kupna – gdy do takiego przebicia dochodziło od góry. Skuteczność systemów transakcyjnych opartych na przecięciu średniej ruchomej i ceny zamknięcia została przetestowana dla 9 różnych rodzajów średnich ruchomych (wykładnicza, prosta, *time series*, trójkątna, *variable*, *volumen adjusted*, liniowo ważona, Hulla i fraktalna adaptacyjna).

Najważniejsze wnioski, jakie zostały sformułowane na bazie przeprowadzonych badań, to:

1. Na polskim rynku kapitałowym najwyższe stopy zwrotu zostały osiągnięte przez średnie ruchome o długości zaledwie kilku sesji. Świadczy to, że na tym rynku dominują transakcje o charakterze spekulacyjnym.

2. Analiza parzystości i nieparzystości długości średnich ruchomych optymalizujących system transakcyjny wykazała, że żadna z nich, tj. parzysta lub nieparzysta, nie była dominująca na polskim rynku.

3. W przypadku żadnej z par średnich ruchomych i dla wszystkich badanych spółek nie miał miejsca przypadek, że zawsze jeden typ średniej ruchomej przynosił wyższe stopy zwrotu niż drugi typ średniej. W przypadku niektórych z takich par odsetek ten był jednak bardzo wysoki, jak np. dla pary średnich ruchomych *T* i *Hull* (i spółek należących do indeksów WIG20 i mWIG40) czy też *Tri* i *Var* (w przypadku spółek z indeksu sWIG80).

4. W przypadku dwóch systemów transakcyjnych opartych na dwóch różnych średnich kroczących można wskazać ten, który dla większej liczby akcji okazał się skuteczniejszy. Na tej podstawie opracowano rankingi efektywności dla analizowanych typów średnich kroczących, w których średnie kroczące sklasyfikowane na najwyższych pozycjach to *time series* i trójkątna. Z kolei na najniższych pozycjach w rankingu uplasowały się następujące średnie kroczące: fraktalna adaptacyjna, Hulla i *variable*.

5. W rankingach stabilności systemów transakcyjnych dla poszczególnych rodzajów średnich kroczących, według miar MM1 i MM2, najbardziej stabilne okazały się systemy bazujące na średnich: ważonej liniowo, wykładniczej i trójkątnej, natomiast najbardziej niestabilne wykorzystywały

wskazania takich średnich, jak *time series* i Hulla. W przypadku miary MM3 wniosek był zgoła odmienny.

6. Warto podkreślić fakt, że z jednej strony systemy transakcyjne wykorzystujące wskazania średniej ruchomej *time series* generują najwyższe stopy zwrotu, ale jednocześnie są najmniej stabilne.

Najważniejszym ograniczeniem badania była różna długość horyzontów inwestycyjnych dla poszczególnych spółek – komponentów indeksów giełdowych. Wybranie długiego horyzontu czasowego miało przełożyć się na bardziej miarodajne wyniki. Z drugiej strony takie podejście utrudnia przeprowadzenie porównań między poszczególnymi spółkami i średnimi.

Podobne badania mogą zostać przeprowadzone dla pozostałych spółek notowanych na GPW w Warszawie, a także dla takich klas aktywów, jak waluty, surowce czy indeksy giełdowe.