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Analysis of the denomination structure of the Polish currency in the context of the launch of the new 500 zloty banknote

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Abstract

Even though cashless payment instruments are proliferating and there is talk of the expected demise of cash, the latter is still widely used, and not only in Poland. The sharp rise in cash in circulation, especially over the last few years, makes it interesting to take a closer look at its most important characteristic – the denomination structure. In this study we analyse this structure with the use of the D-Metric model in the context of NBP's recent decision to introduce the new 500 zloty banknote. A comparative analysis is also performed of the structure of the domestic currency with the structures observed in other countries. We show that the denomination structure of the Polish currency has been almost suitable to the prevailing economic conditions since 2015. The exception is 1 and 2 grosz coins, which – according to our results – should be demonetized. The decision about the introduction of the new higher denomination note seems to be an element of adjusting the denomination structure to the conditions in Poland.

Keywords: denomination structure, D-Metric model, principle least effort, principle least cost, 500 zloty note

JEL: C22, C32, E41, E42, E51

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1 Introduction

One of the tasks of central banks is to issue sufficient stock of banknotes and coins to meet market demand. This also entails modernising from time to time the banknotes' security features. It seems that in the absence of changes in the currency denomination structure in Poland since the redenomination of the zloty in 1995,[1] there is a need to analyse its denomination structure in terms of its correspondence with the prevailing economic conditions. Central banks worldwide not only perform a periodical review of banknotes' security features in the face of ongoing advances in counterfeiting techniques: the central banks' practice also involves the review of parameters determining the currency denomination structure, such as values of the lowest denomination coins and the highest denomination banknotes, the boundary between banknotes and coins and the number of denominations. All of them are analysed in detail in this paper.

An additional parameter should also be distinguished, namely the so-called spacing of denominations, which is analysed, among others, by Wynne (1997), Van Hove (2001) and Tschoegl (2001). This parameter changed, for example, in the Netherlands as a result of the launch of the euro in 2002: the 1–2.5–5 system used in the case of the Dutch guilder was replaced with the 1–2–5 euro system. The Polish currency already follows the 1–2–5 system and there is no need to deal with this problem.

The decision about the modification of the denomination structure by the introduction of 500 zloty notes almost coincided with the debate about the rationality of keeping the highest denominations such as EUR 500, CHF 1,000 and USD 100 in circulation.

To exemplify the emotions surrounding these debates, let us quote Charles Goodhart, who called the European Central Bank and the Swiss National Bank "shameless" for issuing "vastly high--denomination notes", namely the EUR 500 and CHF 1,000, "which are there to finance the drug deals."[2] Another example comes from *The Washington Post*, where Summers (2016), writing and citing a working paper by Sands (2016), extended the indictment to the USD 100 bill: "it too is used by criminals, so let's get rid of it."

Less emotional and more matter-of-fact analyses of the issue can also be found. One of these is certainly the latest report of Europol (2015) entitled unambiguously *Why is cash still king?* Van Hove, Vuchelen (1996b) and Van Hove (2007) suggested that the ECB should place the upper limit of its banknote series at EUR 50 instead of EUR 500.

Among opposite opinions were those voiced, among others, by Antti Heinonen, former head of the ECB's Directorate, Banknotes, who said: "Clearly cash is used by criminals because it is an anonymous instrument. But to say that it would be more difficult to commit a crime if we didn't have high denomination notes would be to confuse cause and effect. If we didn't have the higher denominations, criminals would use the lower denominations – or other global currencies, such as the US dollar or Swiss franc." (Atkins 2006).

A reasonable stance in this matter was taken by, among others, Rogoff (2015), who besides the above-mentioned benefits of withdrawing cash, presented a set of arguments for its preservation:[3]

– the loss of seigniorage;

– the existence of cash ensures diversification of the payment system;

[1] By removing 4 zeros, i.e. 10,000 old zloty (PLZ) = 1 new zloty (PLN).

[2] Quotes available on Bloomberg web (Stirling 2015).

[3] But two years later the point of view of Rogoff (2016) was somewhat radicalized to withdraw cash.

– it is an element of the guarantee of civil liberties;

– the withdrawal of cash in one country entails the risk of using cash from other countries.

It seems that the adversaries of high-denomination banknotes have attained their goal: it has turned out that as of the end of 2018, the ECB intends to discontinue production of EUR 500 notes of the new Europa series. The existing banknote will, of course, continue to act as legal tender.

Given the ongoing debate on the case for maintaining high-denomination notes in circulation, the question arises if the decision to put the 500 zloty note into circulation is justified from the point of view of denomination structure models. Official releases of the central bank mention such reasons behind the decision as a reduction in issuance costs and an increase in efficiency of managing the strategic cash stock held by the bank. Manikowski (2016) mentions an additional reason, namely that the introduction of the 500 zloty note would be an element of the adjustment of the denomination structure of the currency to the economic conditions prevailing in the country.

The main criterion of analysis of the denomination structure should be its optimality (efficiency), which means it not only minimizes the costs of money issuance and management, but also helps to make cash transactions in an efficient way. In other words, the denomination structure is defined as optimal if it ensures the efficiency of payments made by the use of a minimum amount of coins and notes (also called tokens) during transactions. Such a solution involves maintaining and carrying a small stock of physical money, which is related to the cost incurred by stakeholders, called social costs. According to Schmiedel et al. (2012, p. 8) the social cost is related to, "the resource costs incurred by all stakeholders (i.e. consumers, retailers, companies, banks and cash-in-transit companies) in the course of all activities along the payment transaction chain." It is computed by adding up the private costs of all stakeholders and eliminating any transfer payments in order to avoid double counting.

In general, the non-optimal denomination structure may bring about very serious consequences:

– for the central bank because of incurring high costs of the issuance of cash and management of cash reserves (see, e.g. Pattanarangsun 2011),

– for society, due to the high level of social costs (see, e.g. Bouhdaoui 2012),

The issues associated with the structure of denominations are complex and require in-depth research, which should include an analysis of the economic circumstances and factors influencing the demand for cash. Therefore, the aim of this study is to analyse the compatibility of the denomination structure of Polish currency with the economic conditions prevailing in Poland, referring in addition to examples of other countries. To achieve this purpose we conduct the following:

1) a general discussion about the optimality and disadvantages of inappropriate components of the denomination structure;

2) a comparative analysis of models used to design the optimal denomination structure;

3) an analysis of the economic situation in Poland, especially in the era of low interest rates and deflation;

4) an empirical study of the demand for larger and smaller denomination notes using the cointegration framework to identify factors influencing currency demand by denominations in Poland;

5) an analysis of hoarding of larger denomination notes, which is important from the point of view of the introduction of the 500 zloty note;

6) some cross-country comparisons from the denomination structure point of view.

Consequently, the paper is organized as follows. Section 2 presents a discussion about the optimality of the denomination structure and an analysis of the different models allowing a consideration

of the denomination structure, such as Cramer's, Henstch's and D-Metric model. Section 3 discusses the data used in the paper and applies the selected model in the Polish context, assessing the denomination structure. Section 4 contains cross-country comparisons of denomination structures. The last section presents conclusions from the analyses.

2 Optimality and models of currency denomination structure

An inappropriate denomination structure can be the result of too low or too high values of parameters which define this structure. The consequences of its inadequacy indicated by Pattanarangsun (2011) are presented in Table 1 and some selected ones are subjected to critical analysis below, i.e.:

- the phenomenon of lowest denomination coins lost in circulation,
- inefficiency of payment in the case of too high value of the lowest denomination coin,
- the psychological impact of too high banknote denominations on inflation growth,
- the boundary between coins and banknotes,
- inadequate number of currency denominations.

2.1 The phenomenon of lowest denomination coins lost in circulation

This phenomenon, which was observed in the times of the so-called precious metal coins, can be seen in every country (see, e.g. Kippers, Franses 2003). It has been the subject of many detailed analyses, of which the most important include, among others, the book by Sargent and Velde (2012) tracing the phenomenon up to the 12th century, the working document of the European Commission (2013) and articles, among others, by Goldin (1985), Amromin and Chakravorti (2009), and Wood (2011) all addressing the currently observed problem in many countries. The easiest way to explain the current phenomenon of the coins lost in circulation is to analyse the relation of the nominal value and value of material used for coin production. When the former of these values is lower, coins are, among others, remelted and found to be used in industry. In this case, we have to do with a short, sometimes zero lifespan of such coins or low durability which has nothing to do with their physical durability. An important reason for coins being lost in circulation is also their substantial stock gathered by consumers.

2.2 Inefficiency of payment in the case of too high value of the lowest denomination coin

This inefficiency is highly controversial. Many studies have shown that, for example in the case of the euro, from the theoretical point of view (according to Cramer's rule described later) price rounding up to 5 euro cents (thus excluding 1 euro cent and 2 euro cent denominations) actually increases payment efficiency (see e.g. Kippers, Franses 2003). Therefore, the denomination structure without 1 euro cent and 2 euro cent denominations is more efficient. However, the empirical study by Bijwaard and Franses (2009) shows that after price rounding in the Netherlands, the number of 1 and 2 euro

cent coins in the consumer's wallet substantially decreased. But there were inadequate quantities of other coin denominations (too many or too few of them). This may imply that rounding does not make the denomination structure of the euro in a given country more optimal. However, the authors of the cited study indicate that it is necessary to conduct studies in other countries in order to verify their conclusion.

Another example refers to New Zealand, where after the withdrawal of 1 cent and 2 cent denominations in 1988, the demand for a replacement 5 cent coin increased (see, e.g. Aubry, Dupuls, Vachon 2008). Such a situation has far-reaching repercussions in supply logistics and expenses associated with the production and handling of coins. But the average production of 3 small-denomination coins (5 cents, 10 cents and 20 cents) for the period from 1988 to 2004 remained almost unchanged.

2.3 The psychological impact of too high banknote denominations on inflation growth

Some authors argue that the introduction of higher denominations increases the risk of higher inflation as argued, among others, by Chen (1976), and Chen and Tsaur (1983). In the context of deflation observed in Poland since the 3rd quarter of 2014 till the 4th quarter of 2016, this could be deemed a desirable phenomenon, especially amid concerns about a deflationary trap which was defined by Keynes (1936, pp. 103–104); see also Brzoza-Brzezina et al. (2015).

Empirical studies do not confirm Chen's argument. For example, Franses (2006), after analysing the data spanning over a period of 40 years in 59 countries, established the existence of a reverse causality, namely the impact of rising inflation on the decision to introduce a high denomination. Similar conclusions were reached by Egbuna and Obikili (2013), who analysed the impact of the introduction of new denominations on inflation in Nigeria in 1973–2011. It should be noted that they observed a short-lived impact of a rise in food prices each time a new denomination entered circulation. Yet, they explained this by the impact of such decisions on inflation expectations, which were noticeable within a horizon not exceeding one month.

Another example of empirical research comes from the euro area countries. The introduction of the common currency in almost all countries may be, in a sense, treated as an introduction of new, higher denominations. This is particularly true for countries without the equivalent of 200 euro notes or 500 euro notes in particular (e.g. it does not apply to Germany, the Netherlands and, to some extent, Austria where there was a legacy banknote of a similar value as the EUR 500 banknote). Thus, research on the impact of the euro cash changeover on inflation can also be used to identify the effects of the introduction of high denominations. Yet, this kind of research is always subject to some error as the increase in inflation could also have been affected by price rounding.

Nevertheless, certain institutions, including Eurostat (2003), showed a very limited impact of the euro changeover on inflation. Yet, it should be pointed out that a number of studies pointed to significant changes in the level of prices as perceived by consumers, which did not coincide with the official data of national statistical offices (see, e.g. Angelini, Lippi 2006 for Italy; Room, Urke 2014 for Estonia).

To sum up, the above considerations show that the argument that the introduction of high denominations leads to an increase in inflation should be rejected.

2.4 The boundary between coins and banknotes

Most researchers agree that the higher the denomination, the less frequently and with greater care it is used for transaction purposes (apart from its undisputed importance for hoarding), hence its longer lifespan. This was proved by numerous studies. For example, Raghubir and Srivastava (2009) defined the so-called denomination effect which is related to the following consumer behaviour:

– the likelihood of spending a certain amount of money was lower in the case of one banknote than in the case of several lower denomination banknotes, with the same total nominal value,

– consumers deliberately choose to receive money in a large denomination when they need to exert self-control in spending,

– from the psychological point of view, large denominations are less easily replaceable than small denominations.

On the other hand, the lower the denomination, the more frequently and with less care it is used, leading to a shortened lifespan and consequently higher production costs.

According to Mushin (1998) and Auhry et al. (2008), the transition between coins and banknotes should result from the relationship between currency production costs and lifespan. However, an important aspect is often forgotten which should be taken into account in such considerations: the much weaker security features in the case of coins and hence the growing risk of their counterfeiting.

Another criterion was given by Koeze (2006), who claims that the boundary between banknotes and coins should be determined based on the following two characteristics:

– lifespan of a banknote – L,

– the time period between the returns of the banknote to the central bank – C.

A value of $N = L/C$ as low as 1 may indicate that a banknote with these characteristics should be replaced by a coin. The reasons for using this criterion are intuitive. Koeze (2006) notes that if the banknote becomes unfit before being returned to the central bank (i.e. when $N < 1$), then it remains for some time in circulation in a condition unfit for circulation and the central bank should prevent such a situation by replacing the banknote with a coin.

According to own calculations conducted by Manikowski and Oleś (2014), the lifespan of the 10 zloty banknote L is approx. 2 years, and the return time C is approx. 1 year, which means that $N > 1$, so in the case of this denomination the Koeze's criterion of the banknote's replacement with a coin is not met.

We should also highlight a certain aspect related to the circulation of banknotes and coins of the same denomination. The experience of some countries has shown that people prefer banknotes to coins – example of 1 US dollar coin (see, e.g. US Government Accountability Office 2013; Lotz, Rocheteau 2004). Thus, in the case of Poland, during the 1995 redenomination of the zloty it would have been justified to put 2 and 5 zloty notes into circulation instead of coins (leaving aside the production cost aspect), as before the 1995 redenomination coins were practically not used. Poles were "accustomed" to banknotes.

2.5 Inadequate number of currency denominations

Kohli (1988) and Kitamura (1997) observed that the absence of certain denominations may lead to higher demand for neighbouring denominations that exceed the estimated demand for the missing denomination. The best example deals with New Zealand.

The above considerations point out the necessity to define the optimal denomination structure, i.e. the structure with an appropriate value of mentioned components. In the literature we may find several approaches to determining the optimal currency denomination structure. Here we discuss the most popular ones.

The first approach is based on the principle of least effort (PLE), which can be achieved with the smallest number of coins and notes (the pre-defined co-called tokens) used in payment purposes. In other words, the optimal denomination structure is defined as a mix of coins and notes which ensures the efficiency of payments as payments in which the smallest number of tokens are involved. The concept of efficient payment was first introduced by Caianiello, Scarpetta and Simoncelli (1982) and modified by Cramer (1983). In a number of later papers (see, e.g. Boeschoten, Fase 1989; Sumner 1993; Van Hove, Heyndels 1996; Van Hove 2001), this was the starting point for further analyses. Cramer defined the efficient currency denomination structure as the one which requires a minimum number of tokens among all efficient payment models. To solve this issue Cramer (1983) formulated efficient payments in mathematical terms as a solution to an optimizing problem in the following way (according to notation used by Pattanarangsun 2011, p. 167).

Let A means the amount to be paid and $n(A)$ – the amount of notes and coins to be used for cash payment. If the different denominations are numbered as $d = 1,...,D$ with $v(d)$ as a face value, then $n(A, d)$ and $\sum_{d=1}^{D}\left|n(A,d)\times v(d)\right|d$ denote the number of tokens of denomination d and value of tokens of all denominations respectively, used for paying amount A. A positive $n(A, d)$ refers to use as a payment, while a negative $n(A, d)$ means that the n tokens of denomination d are given as change. Efficient payments $n(A, d)$ are then the solution to the following problem:

Minimize $\quad n(A)=\sum_{d=1}^{D}\left|n(A,d)\right|$

with constraints $\quad \sum_{d=1}^{D} n(A,d)\times v(d)=A$

The second approach is based on the idea that the number of denominations should be minimized. The smaller the number of denominations, the lower the issuing costs, and the easier it is for the public to recognize the different notes and coins for cash transactions. Telser (1995) followed by Wynne (1997) and Tschoegl (1997) observed a similarity between the problem of determining the optimal form of denomination structure and the optimal set of weights called Bachet's weights problem,[4] i.e. what is the smallest number of standard weights needed for weighing any quantity within a given interval on a two-pan balance where the standard weights correspond to denominations? The number of denominations is minimized in this problem, rather than the number of tokens exchanged.

[4] Bachet's weights problem comes from the French mathematician Cluade-Gaspard Bachet de Meziriac (1581–1638). It consists in finding the minimum number of pound weights that can be used to weigh goods with any integral number of pounds from 1 to 40. According to the solution given by Bachet in 1624, there should be 4 weights of 1, 3, 9 and 27 pounds.

Van Hove (2001) proved that finding the optimal range of denominations is not the same as Bachet's problem solution. Additionally, Van Hove (2001) argued that determination of the optimum range of currency denominations is a multi-criteria optimization problem, where the greatest weight should be assigned to the criterion based on the principle of least effort (PLE).

The PLE states that:

– it is more convenient for transactors given that it reduces the bulk and weight carried around by the cash-using public in turn limiting handling costs,

– it keeps down the number of coins and notes in circulation and thus the production costs incurred by the central bank.

Following the second argument, it is therefore preferable for the central bank to opt for a currency system that limits the number of coins and notes used in transactions. However Bouhdaoui and Bounie (2012) demonstrate that this argument is biased and that efficient payments increase the production costs incurred by the central bank. They proved, on the example of the US currency in 2010, that the use of the PLE principle may, in the case of a great diversity in the costs of production of specific currency denominations, boost the cost of cash issuance. To this end, by analogy to the PLE, they developed a new model based on the principle of least cost (PLC). Bouhdaoui and Bounie (2012) have shown on the example of the US currency that although the use of the PLE minimizes the number of tokens, it increases the production costs of used currency denominations by an average of 24.2% as compared to the cost criterion of the PLC.

The approaches mentioned above belong to the group of theoretical methods which are based on optimization. There are also practical (empirical) methods to determine the adequate denomination structure: Hentsch's and the D-Metric models without a theoretical basis.

Hentsch (1973, 1975) observed the existence of a proportional relationship between the amount in circulation for a denomination and the square root of its nominal value in the case of a series based on fractional-decimal triples (i.e. 1–2.5–5–10–25–50–…).[5] He defined an indicator of corrected circulation (CC) constructed as the amount in circulation per each denomination (V_i) divided by the square root of the nominal value (v_i):

$$CC_i = \frac{V_i}{\sqrt{v_i}}$$

Then, Hentsch (1973, 1975) claims that if the CC_i calculated for an i denomination was considerably higher than the values of CC calculated for other denominations, there could be a need for a higher denomination. Conversely, if the CC_i is lower, this suggests that it would be reasonable to withdraw this denomination.

This method was used, among others, by the European Commission (2013) to analyse the plausibility of the withdrawal from circulation of 1 and 2 euro cent coins.

It should be noted that the review of the literature, which uses the Henstch approach, shows a lack of consistency in defining V_i, namely the amount of i-th denomination in circulation. In some cases, V_i means value (the original studies by Hentsch (1973, 1975) and by Van Hove, Vuchelen (1996a) indicate the value approach), while in other cases the volume of banknotes with denomination i (e.g. in the case of banknote analysis – Nenovsky, Hristov 2000, in euro analyses – European Commission 2013).

[5] The series based on binary-decimal triples (1–2–5–10–20–50–100–…) are not the same, because between 2 and 5, 20 and 50 the ratio is 2.5, not 2.

Another empirical approach uses the so-called D-Metric model, which was proposed by Payne and Morgan (1981). This model assumes that specific denominations of banknotes and coins, due to inflation, reduce their purchasing power. The authors have noted that from the point of view of the denomination structure, the change in the purchasing power of cash is best described by the average day's pay D which means average daily wage level.

Additionally, taking into account a day's pay allows to consider all motives of cash use: transaction, precautionary and speculative.

Moreover, studies conducted in 35 countries on 1979 data have shown that this amount is sufficient to describe the denomination structure of currency in circulation. In these studies, Payne and Morgan (1981) have found that there are only 3 rather insignificant exceptions: Germany, France and Argentina. In the case of Germany and France, coins with the face value of DEM 5 and FRF 10 respectively were in the range of banknotes. For these countries, the model boundary was 4.9 and 9.8 respectively, thus close to the actual boundary. After a short period, the structure of circulation coincided with the model structure. In Argentina, the ARP 500 banknote was in the range of coins. The authors of the model explain that the Argentinian government was late with replacing the banknote with a coin.

The idea of the D-Metric model is presented in Figure 1. As Figure 1 shows, the model assumes the existence of 6 coin denominations and 6 banknote denominations and 1 denomination at the boundary between coins and banknotes. The exact ranges of particular currency denominations are based on the previously mentioned binary-decimal triplets used in many countries. Analysis of the denomination structure of a wide range of countries and the average wage by Payne and Morgan (1981) reveals the following pattern between the value of D and the denomination structure:

– the coins-banknotes boundary is located between D/50 and D/20,

– the smallest useful coin denomination should not be lower than D/5000,

– the highest useful banknote denomination should not be higher than 5D.

Because of its simplicity and lack of theoretical basis, the D-Metric model is exposed to criticism. The most well-known criticism is presented by Mushin (1998) in reaction to Barry (1994). For example he states that payment habits, cultural factors and wealth-holding have an impact on denomination structure and can differ both between countries and over time. Mushin (1998) also expresses his doubts mentioned and analysed earlier about the note-coin boundary.

We may add further doubts about the correctness of the D-Metric model which deal with the following:

– taking into account the development of cashless payment instruments,

– validity at the present time,

– the optimality of D-Metric model's denomination structure from the social cost point of view.

As to the first doubt, it seems that cashless payments instruments may influence the volume of different denominations without affecting the range of denominations. For example, the increasing use of payment cards does not eliminate the amount of bills to pay in cash. In that case we will continue to use cash but maybe in smaller volume. Of course, this remark requires empirical verification.

The second doubt can be dispelled by a similar analysis to Payne's study, which has been made in a limited version in Section 4. The last doubt is related to the question of whether optimality in the PLE sense is the same as the rationality of participants of cash transactions. In other words, does the public use cash in an efficient (optimal) way? To answer the question, a survey of households is needed.

The D-Metric model, despite its simplicity and criticism, has been successfully used in practice in many countries to analyse the existing denomination structures of cash in circulation (Crickett 2012).

The best known examples of applications of the D-Metric model are carried out in New Zealand and Canada. Barry (1994) conducted an analysis of the currency denomination structure in New Zealand with the use of this model and showed the rationality of the following decisions:

– entering a new denomination (NZD 50) banknote in circulation,

– withdrawal from circulation of 1 cent and 2 cent coins in 1987,

– replacement of NZD 1 and NZD 2 banknotes with coins of the same face value in 1991.

Interestingly, in New Zealand the 5 cent coin was also withdrawn from circulation in 2006.

In Canada, the D-Metric model provided the basis for the following decisions (see Aubry, Dupuls, Vachon 2008):

– withdrawal of the 1 cent coin from circulation,

– replacement of 1 dollar and 2 dollars banknotes with coins of the same face value in periods which coincided with those suggested by the D-Metric model.

The use of the D-Metric model for the analysis of the currency denomination structure is also discussed by Abrams (1995). Another example comes from Jamaica, whose central bank announces on its website that the D-Metric model is the primary tool used by it to determine the denomination structure of the national currency in circulation. The Bank of Jamaica lists additional conditions to be met by the currency denomination structure. Thus, for example, the greatest currency denomination should not account for more than 60% of the value of currency in circulation. If this threshold is achieved or exceeded, a higher denomination banknote should be put into circulation (see http://www.boj.org.jm/currency/currency_policy.php). The D-Metric model has recently been used in the Maldives by Fahmy (2016).

The D-Metric model is based on the empirically observed relationship between the value of D and the currency denomination structure. However, the literature shows considerable differences in determining the value of D. For instance, Payne and Morgan (1981) used the data from reports published every three years by the Union Bank of Switzerland as the basis for determining the value of D. The reports compare prices and wages in selected cities around the world based on surveys conducted among employees of 15 professions. However, the analyses of the currency denomination structure in Thailand by Pattanarangsun (2011), and in the Maldives by Fahmy (2016), adopt the daily GDP per capita as their basis. The research conducted by Mougeot and Comm (1997) in Canada took into account the daily value of credit card transactions. On the other hand, in 2008 the total nominal labour income, net of personal transfer payments made to the government, was adopted by Aubry, Dupuls and Vachon (2008) as the basis for determining the value of D.

Another problem is related to the country's population and the number of days in the year – the next data necessary to determine the value of D. For example, in Thailand, the total number of working days in the year and the country's entire population was taken into account. However, in Canada, the number of working days and employment were considered.

It should also be noted that Payne and Morgan (1981), while analysing the denomination structure of cash in circulation in the United Kingdom, noticed an interesting correlation in the relationship between the boundary of coin and banknote and the amount of currency in circulation. They found that the amount of denominations of currency in circulation, classified by the D-Metric model as

banknotes, was approximately the same in 1972–1979. But in our opinion the use of the model to determine the optimal amount of banknotes in circulation is doubtful.

To conclude the discussion of denomination structure models, it should be pointed out that many economists agree that in order to determine the optimal denomination structure, it is desirable to minimize the total cost and maximize, at the same time, the utility explained by the demand for cash and high efficiency of transactions according to the principle of least effort. When doing so, the following should be taken into account:

– the demand side, including the two parties to the transaction and commercial banks in terms of utility and efficiency of payment,

– the supply side, including the central bank and commercial banks from the costs point of view.

The analyses presented in this article, use the D-Metric model. The reasons are the following:

– relatively few problems with the ambiguity of D (opposite to Hentsch's approaches),

– universal use in the practice of central banks,

– possibility of making cross-country comparisons of denomination structures (because of data availability in reports of the Union Bank of Switzerland),

– taking into account transaction, precautionary and speculative motives (opposite to PLE in which only transaction motives are considered).

However, due to some of the mentioned shortcomings of the model and lack of consideration of volumes, the D-Metric model should be treated only as the first step in the denomination structure analysis.

3 Empirical analysis of the currency denomination structure in Poland

3.1 Conditions in Poland

An analysis of the denomination structure in Poland requires a prior characterization of the economic conditions prevailing in the country which, as suggested by the above discussion, are likely to have a significant impact on this structure. This has a particular importance when the decision is made to put a new, higher denomination banknote into circulation.

According to among others Sriram (1999) and Friedman (1969), the demand for cash depends on many factors:

– economic activity usually expressed by GDP or final consumption expenditures of households;

– the opportunity cost of money, which consists of the rate of return on assets representing an alternative to cash (outside money), as well as the rate of return on investment in money (own-rate). For the physical money own-rate is named by Serletis (2007, p. 131) as an implicit rate – a positive rate of return/benefit in the form of services or reduced transaction costs. Various measures of alternative costs were discussed, among others, by Ericsson (1998);

– inflation as a rate of return on investment in tangible assets (also sometimes used to determine the previously mentioned alternative costs; see Ericsson 1998).

Another factor influencing the demand for cash is strictly related to cashless societies. In this case, financial innovation can be considered by, for example, the number of ATMs, bankcards or EFTPOS terminals, or by specific functional forms of econometrics model (described later).

Additionally, in the case of open economies, it is suggested to take into account the currency exchange rates. Many econometric studies conducted in different countries have shown the significance of the above factors (e.g. Sriram 1999). Some concern cash, others – various monetary aggregates starting with the M1 aggregate and ending with the M5 aggregate. Most of these studies, however, concerned a situation in which neither low interest rates nor deflation were observed. Precisely such conditions were observed in Poland from the 3rd quarter of 2014 till the 4th quarter of 2016, when inflation stayed in the negative territory for the first time.

Relatively low interest rates[6] and deflation with growing GDP support an unexceptionally large increase in the amount of currency in circulation (CIC) in relation to GDP observed in Poland (see Figure 2).

The large increase in CIC was highlighted, among others, by Briglevics and Schuh (2014), who show high sensitivity of the domestic stock of currency in circulation to changes in low nominal interest rates in the US. Interestingly, a similar mechanism is observed in Japan, which has struggled for a long time with low interest rates and deflation. According to Amromin and Chakravorti (2009), the rise in value of the CIC to GDP ratio from 9.2% to 16.3% in 1998–2003 was driven by low or negative inflation. However, in the case of Japan, Rogoff, Giavazzi and Schneider (1998) point out additional causes behind the rise in quantity of currency in circulation: a low crime rate, widespread use of cash by the Japanese for transaction purposes, low popularity of payment cards and the high rate of substitution of new banknotes for old ones because of wear out due to soil, stains etc.

In the light of the foregoing deliberations, we should ask about the precise reasons for the growth in currency in circulation in the era of low interest rates and deflation,[7] namely which of the following reasons – transaction, precautionary or speculative function – is the major factor behind this growth.

It seems that the answer to this question should follow an analysis of the circulation of the highest denomination banknote – 200 zloty. Its growth measured by the compound average growth rate (CAGR) for 2010–2015 was 10.08%, reaching 20.10% in 2015.

This banknote, until recently, could be regarded as a value-storing banknote, usually obtained in banks' cash desks or through cash payments of wages. In recent years it is more and more common in ATMs, becoming, apart from the 50 and 100 zloty notes, an ATM banknote.[8] This is confirmed by the daily practice of ATM users as well as NBP data, on the basis of which the average value of a single payment has been determined. Since 2005, this value has increased from PLN 301 to PLN 415 in 2015.[9] As Figure 3 shows, in this period the value of the 200 zloty banknotes in circulation increased 2.6 times, which is almost the same as in the case of a 100 zloty note, which can be also classified as a value-storing banknote.

To analyse the sources of growth in demand for the 200 zloty banknote, we use the vector error correction model (VECM). Additionally, the same approach is used in the case of 10 and 20 zloty. The reason to take into account the small denominations is as follows: if demand for the large value

[6] According to, for example, Rzońca (2014, p. 19) a low interest rate is below 2%.

[7] In general, according to the author, the increase in cash in Poland during the 2014–2015 period is related to some aspect of the liquidity trap.

[8] In cash demand analyses, banknotes are often divided into 3 groups of the following: (1) transaction, (2) value-storing and (3) the so-called ATM banknotes. These groups are not always disjointed. In the case of Poland, the three above-mentioned groups are comprised of 10–50 zloty notes (1st group), 100–200 zloty notes (2nd group) and 50–200 (3rd group).

[9] Calculation based on NBP data on volume and value of ATM withdrawals published on the website http://www.nbp.pl/home.aspx?f=/systemplatniczy/karty_platnicze.html.

note is different from the demand for the small denominations, this should be reflected in either different coefficients on the same variables or different variables determining the demand functions.

The decision about the use of the VECM approach by Johansen and Jesulius (1990) and Johansen (1995) resulted from the proven usefulness of this model to describe the demand for money in the case of non-stationarity time-series. This approach seems to be particularly suitable for verifying the long--run equilibrium (cointegration) relationships on which the theoretical considerations are based.

For example, Doyle (2000) estimated foreign demand for the US, German and Swiss currency with the use of the cointegration framework. He used many ways to estimate cointegrated relationships with, among others, the Johansen procedure. Khamis and Leone (2001) analyse demand for currency under a financial crisis in the case of Mexico with the use of VECM according to the Johansen procedure. Nachane et al. (2013) identify various factors influencing currency demand in India in a vector error correction framework.

Bartzsch, Seitz and Setzer (2015) use the VECM for demand for small and large denomination notes (but in the case of medium denominations setup a VECM is not possible). Noteworthy is the analysis by Belke and Czudaj (2010), who make a comparison of the cointegration VAR and single equation techniques (similar to Dole 2000). They take into account single equation methods like the ARDL (autoregressive distributed lag) approach, FM-OLS (fully-modified ordinary least squares), CCR (canonical cointegration regression) and DOLS (dynamic OLS) and compared them with the commonly used cointegrated Johansen VAR framework. We may also find the use of the VECM in Seitz and Setzer (2009) and Fischer, Köhler and Seitz (2004). The overview by Bondt (2009) of empirical studies on the euro area money demand also implies the use of the error correction framework.

To analyse the demand for denominations we decided to use unseasonal quarterly data[10] from 4th quarter of 2000 to the 4th quarter of 2015 with the following set of variables: the real value of the largest and smallest denominations in circulation, real GDP (both deflated by CPI), WIBOR 3M and CPI. The selection of factors is motivated both by considerations on the empirical money demand models and the availability of forecasts. Their projection is prepared quarterly by NBP for, among others, the Monetary Policy Council. Financial innovation are considered differently. Many authors use direct measures. For example, Vale (2015) takes into account the number of terminals for electronic funds transfer at POS, Aastveit (2005) compiles the number of ATM's and EFTPOS terminals. A similar set of variables is used by Rinaldi (2001) with the number of ATM's, bankcards and EFTPOS. The number of card payments is preferred by Bartzsh, Seitz and Setzer (2015). In order to capture the increasing preference for cashless payments, we use the approach implemented by Snellman, Vesela and Humphrey (2000) and Fischer, Köhler and Seitz (2004) that the financial-innovation--effect may be captured indirectly by including a linear time trend in the regression. Consequently, the cointegration specification for the demand for notes assumes a linear trend in the data in that an unrestricted constant is included. In other words, we assume that the level data have linear trends, but the cointegrating equations have only intercepts.

Like Bartzsch and Seitz (2015), we also experimented with the value of card cashless payments as an exogenous, non-modelled variable. However, we obtained results with non-expected sign and value of parameters in cointegration space.

[10] Ericsson, Hendry and Tran (1994) show theoretically and empirically within the Johansen framework that the number of cointegrating vectors and the cointegrating vectors themselves are invariant to the use of seasonally adjusted or unadjusted data.

Additionally, we use a dummy variable representing a shift in the demand for 200 zloty notes as a result of the collapse of the Lehman Brothers in the 4th quarter of 2008 and seasonal dummy variables. Both kinds of dummy variables are strictly exogenous.

Before the estimation of VECM, a unit root analysis was conducted by augmented-Dickey-Fuller (ADF) and Philips-Perron (PP) tests for the null of the unit root, and the Kwiatkowski-Phillips-Schmidt--Shin (KPSS) test for the null of stationarity. Table 2 presents the obtained results. In the case of the ADF and PP tests, the table shows the results of the last steps of the procedure proposed by Dolado, Jenkinson and Sosvilla-Rivero (1990) for the testing of unit roots in the presence of possible trends. The reason for the use of this procedure is related to the observation that neither the ADF nor the PP test is capable of distinguishing a stationary process around a linear trend from a process with a unit root and drift. A rejection of the unit root null is very unlikely in this case.

According to all the tests, small and large notes in circulation and GDP appear to be I(1), except for a variable of amount of small notes, which according to the ADF test is I(2). In the case of inflation and the interest rate, the ADF and the PP tests yield ambiguous results, but the KPSS test shows its difference-stationarity (i.e. I(1)).

Given I(1) of all variables, cointegration techniques as suggested by Johansen (1995, 2000) are employed.

The first step is then to test the number of lags to be included in the VECM. χ^2 (Wald)-lag exclusion tests in the VECM, the multivariate Portmanteau test for autocorrelation and the multivariate residual autocorrelation LM test suggest that 2 lags in first differences should be included. This lag length is enough to ensure uncorrelated residuals.

As a next step, the number of cointegration relations has to be verified by determining the cointegration rank with the trace test and the maximum eigenvalue test. Because of a small sample bias, we corrected the test statistics according to the proposition of Reimers (1992). Both test statistics strongly suggest the existence of only one cointegrating vector among the variables.

Using the procedure developed by Johansen (1995, 2000), we estimate the log-log demand function for the 200 zloty banknote and for small notes (i.e. 10 and 20). The adoption of the log-log model is indicated from the previously described observations of circulation in Poland, the United States and Japan in the environment of low interest rates. The observations imply that in calculations it is necessary to adopt the function form of the log-log econometric model suggested by Lucas (2000), which is in opposition to the views of Ireland (2009) who suggests using the semi-log model.

Table 3 displays the estimation results of the long-run equilibrium of VECM. We do not show the short-run coefficients of the lagged endogenous variables and coefficients of the exogenous variables.

The signs in the cointegrating equations are as expected:

1 The demand for large and small denomination banknotes rises when GDP increases. The long--run elasticity for 200 zloty equals 1.34, which is significantly greater than 1, resulting from the quantity theory of money and for 10–20 zloty equals 0.53, which is not significantly different from 0.5, resulting from the inventory transaction model developed by Baumol (1952) and Tobin (1956).

2 The demand for large and small denomination banknotes rises when the inflation rate declines. Generally, inflation is the representation of lost benefits from investment in tangible assets. In this situation, this rate constitutes the cost of lost opportunities in connection with investments in non--cash money.

3 The demand for large denomination banknotes rises when the interest rate increases. It seems that the WIBOR 3M may be interpreted as a proxy variable of an own-rate. In the case of smaller denominations, the WIBOR 3M rate turned out to be insignificant and was eliminated from the model.

The adjustment to the long-run equilibrium is reflected in the error correction term. In the case of large denominations, about 16.97% of the imbalance is corrected in one quarter. For small denominations there is a 2.17% correction in one quarter.

An analysis of the adjustment parameters by LR test indicates weak exogeneity of GDP, WIBOR and inflation. The statistical fit of the system of equations is good with an adjusted R^2 of 0.93 for the 200 zloty note and 0.97 for 10–20 zloty notes.

The conclusions drawn from the obtained results, particularly in the context of the long-run income elasticity, are in line with expectations and show that a 200 zloty banknote is predominantly used for hoarding purposes,[11] whereas 10 and 20 zloty notes are primarily used for transaction purposes.

It can also be said that these currency denominations (as well as other ones) increasingly play an additional precautionary function, especially visible in the environment of low interest rates. This is reflected in the fact that, despite the observed increase in non-cash transactions, banknotes are stored in the wallet as a kind of cushion against a situation where, for various reasons, it is not possible to use a card.

Hoarding is the primary function of the highest denomination banknote. However, the exact amount of 200 zloty notes which are used by the public as a store of value is not known and can only be estimated. For this purpose, some methods have been developed, including:

– the return frequency ratio method,

– the method based on the currency denomination structure,

– the method using the lifespan of banknotes.

The first method is based on the return frequency ratio (RFR) which is used by many central banks to measure their level of involvement in the banknote cycle. The indicator shows how often banknotes in circulation flow back to the central bank within a 1 year period (see e.g. DB 2011). The RFR method assumes that one of the main reasons for fewer returns of high denomination banknotes to the central bank is their value-storing function (see Boeschoten 1992, p. 109). If it was not for this function, the return rate of these notes would be at the same level as in the case of lower denomination banknotes. Based on the difference in the values of those indicators (their levels for particular denominations are presented in Figure 4) we estimated the value-storing quantity of 100 and 200 zloty notes. It was assumed that the point of reference are the so-called transaction notes, including 10–50 zloty banknotes.

As shown in Figure 4, for 10, 20 and 50 zloty denominations RFR are greater than for 100 and 200 zloty notes, which is consistent with our expectation that the latter denominations are used for hoarding.

The second method, developed by Kołodziej and Manikowski (2016), is based on the difference in the denomination structure of banknotes in the circulation and composition of cash flow (in term of denominations) returned to the central bank.[12] Because of the fact that high denomination banknotes are partly stored as value, they are the less liquid part of banknotes in circulation, which

[11] According to Boeschoten and Fase (1992), the main reasons behind the demand for high currency denominations include the following: (1) liquidity or availability of money in every situation (according to Keynes, a precautionary motive); (2) security (e.g. important reason in the case of a bank's bankruptcy), (3) privacy.

[12] All data needed for an evaluation of the share of each denomination in circulation and in input cash flow are collected in every central bank.

is not visible in the structure of returned cash flow. The comparative analysis of the denomination structure of currency in circulation and denominations' composition of the cash flow for 2015 is presented in Figure 5.

For example, the share of value of the 10 zloty note in cash flow returned to NBP equals 0.94% and is similar to the share of this denomination in circulation (1.01%). This means that 10 zloty notes are not used as a store of value. It is opposite to 200 zloty notes, which have a greater share in circulation (23.94%) than in input flow (13.89%) as a result of the hoarding function of this denomination.

The last method, developed by Anderson (1977), used and improved later by other authors (e.g. Sumner 1990; Boeschoten 1992; Van Hove, Vuchelen 1996b), is based on differences in the lifespan of banknotes of various denominations. This method, found by Boeschoten (1992) as the most appropriate, assumes that high denomination notes wear out slower, which usually results in their longer lifespan. Factors affecting the above are as follows:

1 High denomination banknotes represent a higher value, so consumers care about them more (as a consequence of the mentioned earlier the denomination effect).

2 High denomination banknotes are partly stored as value and therefore not used in transactions.

As the impact of the second factor only is discussed, Boeschoten (1992) suggests correcting the real lifespan of the banknote in order to obtain the so-called normal lifespan, which is independent of the first factor. For this purpose, he suggests using the model of Laurent (1974) in the following manner. He takes into account that the average life of the smaller notes is not constant, neither over time, nor over the value of the denominations by means of the mentioned semi-loglinear relationship:

$$AL_t = \alpha + \beta \times \ln RD_t$$

where AL means average note life and RD – real value of denomination.

Boeschoten (1992) estimates the above model by using data on smaller notes and next he uses this to evaluate the normal lifespan (*NAL*) for higher denomination. The difference between the normal and actual lifespan allows to calculate the percentage of notes in circulation kept in hoards (*HP*):

$$HP_{it} = \frac{AL_{it} - NAL_{it}}{AL_{it}} \times 100\%$$

Using a similar approach, we estimated the parameters of Laurent's model for transactional denominations 10–50 zloty (see Table 4). The results indicate that a 100% increase in the real value of a note leads to an increment in its average life of 0.92 years.

Boeschoten (1992) obtained the model with the adjusted determination coefficient at the level of 0.05 and the Durbin-Watson statistic equal to 0.52. In our case of 10–50 zloty denominations, these values were 0.51 and 1.67 respectively, demonstrating a considerably better quality of the model.

Then we used Laurent's model to calculate the normal lifespan for 100 and 200 zloty denominations. The results of the analyses with the normal lifespan for each denomination are depicted in Figure 6. Significant differences can be noticed between the real and normal lifespan for 100 and 200 zloty notes.

The hoarding amount of 100 and 200 zloty banknotes in the 2011–2015 period, obtained by using the above described methods, are shown in Table 5 as a percentage of the particular denomination in circulation, and in Table 6 in terms of value.

These estimations indicate that in 2015, 46.88–79.50% of 100 zloty banknotes and 68.56–74.45% of 200 zloty banknotes in circulation were used for hoarding purposes.[13] This represented 47.35–69.70% of the value of all banknotes in circulation, where the share of a 200 zloty banknote accounted for 15.89–16.36%. This share has decreased by approx. 1 percentage point since 2011.

The presented analysis shows that the large notes are primarily used for hoarding. We cannot rule out that the use of a large denomination results from a precautionary effect. Consequently, the velocity and deterioration of these notes are lower than in the case of so-called transaction notes such as 10 and 20 zloty. Additionally, the introduction of the higher denomination can significantly reduce the volume of hoarding notes by the replacement of notes with a lower face value. However, it is not excluded at this stage of analysis that it could facilitate operations in the shadow economy.

3.2 Analysis of the currency denomination structure in Poland

The analyses of the Polish currency denomination structure relies on the D-Metric model. In order to achieve comparability with the studies by the authors of the model, the data from reports of the Union Bank of Switzerland (UBS) were used. The studies cover the period of 2000–2015. The data contained in the UBS report are related to capital cities (including Warsaw). In order to obtain the results for the whole country, wages in the capital were revised downward to the national average using GUS (Central Statistical Office of Poland) data (the relation of average wages in Warsaw to average wages in Poland). The data used in the analyses, derived from UBS reports, are shown in Table 7.

Relying on the data from Table 7 and on GUS data, the average wages for Warsaw were adjusted to obtain the average wages for the whole of Poland. The results are presented in Table 8.

Then the D value was determined, taking into account the number of days from Table 7. The results are presented in Table 9, which also shows other characteristics of the D-Metric model.

Based on these characteristics, the currency denomination structures for 2000 and 2015 are developed using the D-Metric model. Table 10 shows that in 2000 the appropriate currency denomination structure was comprised of 1 grosz – 100 zloty denominations (i.e. without the 200 zloty banknote) with banknotes starting with 2 zloty.[14] Likewise, in 2015 the theoretical denomination structure consisted of 5 groszy to 500 zloty denominations (i.e. without 1 and 2 grosz coins), with banknotes starting from 10 zloty denomination.

Figure 7 illustrates changes in the denomination structure of currency in circulation from 2000 to 2015. During this period, aggregate inflation stood at 56.20%, whereas in the period from 1995 to 2015 the CPI rose by 229.90% (according to GUS data).

[13] We may distinguish temporal (e.g. in ATM) and permanent (e.g. in sock) hoarding.

[14] Interestingly, in 1990 as part of the zloty redenomination process – originally scheduled for 1992 – a new series of 9 denominations of banknotes was prepared. The series of banknotes called "Cities of Poland" was designed by Waldemar Andrzejewski, and their denominations ranged from 1 zloty to 500 zloty. However, they did not enter circulation.

The results indicate that over the 15-year period higher denominations took over the functions of lower denominations. During this period, the D-Metric model suggests that 1 grosz and 2 grosz denominations should have been withdrawn and that it is legitimate to have in circulation 200 zloty and 500 zloty banknotes.

4 Cross-country comparisons of currency denomination structure

When analysing the denomination structure of the Polish currency, it seems useful to broaden the discussion with some cross-country comparisons. The countries have been selected on the basis of GDP per capita, which results from the quantity theory of money, according to which GDP per capita is one of the main factors affecting the amount of currency in circulation.

In order to perform a comparative analysis, the euro area and 7 European non-euro area countries were taken into consideration. Figure 8 presents GDP per capita and the CIC to GDP ratio.

For the selected countries, we analysed the currency denomination structure, presenting denominations of particular banknotes in zloty calculated according to the PPP – purchasing power parity (Table 11). It also shows the case of Poland without and with 500 zloty banknote.

It should be noted that only in three countries does the value of the highest denomination not exceed 200 zlotys. However, in Romania, the Czech Republic, the euro area and additionally Switzerland, respective values exceed 500 zlotys.

The number of banknote denominations should also be highlighted. With the exception of Sweden, the UK, Norway and Poland (without 500 zloty), all countries have 6–7 denominations, i.e. the number indicated by the D-Metric model.

In order to conduct a cross-country analysis of the currency denomination structures, we choose Bulgaria, Hungary, Romania and the Czech Republic because of the similarity of GDP per capita and CIC/GDP to Poland. The UK was taken into account because the share of the highest denominations of notes in circulation is similar to the share of such denominations in Polish circulation. Additionally, one non-European country – Israel – has been included for the following reasons:

– the PLN/ILS exchange rate fluctuates around 1,
– the banknote denomination structure is similar,
– there was plan to put into circulation a banknote with the face value of 500.

The obtained results are presented in Table 12.

In the case of almost all the analysed European countries, the slot for the highest denomination has been “filled”. Only in the case of Poland and the UK does this place remain blank. Although Scottish and Northern Ireland denomination structures include GBP 100 notes which are legal currency there and can be accepted throughout the United Kingdom (but are not classified as legal tender anywhere in the UK). A lack of middle denomination may be noticed in Romania. Interestingly, the analysis of the denomination structure of the Israel currency points to the theoretical possibility of not only a 500 shekel banknote, but also a higher denomination, namely a 1,000 shekel banknote being in circulation.

Due to the high similarity between Poland and Hungary in terms of GDP per capita, we analyse the denomination structure of the Hungarian forint in more detail. Namely, it exhibits exceptional flexibility, which manifests itself by frequent changes in the denomination structure in response to the changing market conditions. The D-Metric model for this country (see Table 12) points to the lack

of one, lowest denomination coin (2 forint) in 2015. However, this is a consequence of the 1 March 2008 withdrawal from circulation of 1 forint and 2 forint coins. According to Leszko (2009), the most important reasons for such a decision are as follows:

– a lack of active use of low denominations in cash payments,

– production cost exceeds the nominal values 5–6 times,

– cash related costs (circulation, transportation, processing, storage etc.) can account for up to 0.5–0.6% of GDP.

It seems that the withdrawal of the 2 forint denomination will ensure compliance of the real currency denomination structure with the model in the near future.

The analysis of the D-Metric model shows that the highest coin denomination (200 forint) lies at the note and coin boundary (D/50, D/20). This is the result of the decision about the replacement of the 200 forint note with a coin in 2009.[15] Hungary's central bank took into account the incurred cost of production, which in the case of the analysed 200 forint banknote accounted for 17% of production costs of all banknotes (see Szucs 2008).

5 Conclusions

Despite "prophecies" which for many years have been heralding the forthcoming decline of cash usage by the public or even its total demise, cash has been and will continue to be used as a payment instrument in many countries.

It seems that as a result of the development of non-cash payment instruments at normal interest rate levels, which will soon be observed, we may expect cash to grow steadily with an increase in the efficiency of the cash cycle. This means that an analysis of money and the currency denomination structure seems necessary both from the point of view of the central bank and the consumer. This article can be treated as the first comprehensive approach to this complex, yet largely underestimated issue.

While the analyses presented in this article have been performed using a relatively simple D-Metric model, its popularity among central banks and empirical verification in many countries allows us to draw several important conclusions:

– from the denomination structure point of view, redenomination of the Polish zloty in 1995 did not completely take into account prevailing economic conditions at that time;

– according to the model approach, the present denomination structure of the Polish currency is beginning to suit the existing market conditions;

– 1 and 2 grosz denominations, needed in 2000, should now be demonetized, i.e. withdrawn from circulation;

– analysis of the denomination structure and of demand for 200 zloty notes supports NBP's decision to introduce a new, higher denomination banknote; however, it is advisable to include in the future econometrics analysis the demand for 200 zloty notes from abroad due to increasing immigration for work purposes (e.g. from Ukraine);

[15] Hungary has a much richer history of replacing banknotes with coins. The changes concern the 20 forint note which was replaced with a coin in 1992, the 50 forint note replaced in 1995 and the 100 forint note replaced in 1998.

– experience of many countries proves that the risk of a growth in inflation as result of the introduction of the higher denomination seems not to have empirical support in the case of Poland;

– the impact of the introduction of the 500 zloty note on the shadow economy remains an open question. In our opinion this issue requires further research in the general context of the use of cash in the informal economy.

In addition, the D-Metric model can be used to predict a situation requiring an adjustment of the currency denomination structure in Poland to match it to the prevailing market conditions. For this purpose, it is necessary to forecast the D variable.

Of course, there are different, yet rather occasional critical opinions on the simplicity with which the D-Metric model describes the currency denomination structure using only one variable. It is precisely this simplicity that should speak in favour of using this model as the first step in our analysis. It seems that the next step should be based on the principles described in this article: the principle of least effort (PLE) and the principle of least cost (PLC). However, these methods take into account only the transaction motive. It is this objective that the author is committed to pursue.

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Appendix

Table 1

Consequences of the mismatch of denomination structure of currency in circulation

Element of the structure	Too high value	Too low value
The lowest denomination (coins)	– inefficient payment according to the principle of least effort[1] – potential impact on inflation from price rounding up	– difficult to calculate – hoarding phenomenon – tend to generate negative seigniorage
The highest denomination (banknotes)	– psychological impact on inflation growth – risk of illegal transactions and growth in counterfeiting	– inefficient payment according to the principle of least effort – inconvenience in the case of large payments
Transition between coins and banknotes	– high production costs from producing the highest denomination coins (more costly at low demand)	– high production costs from producing the lowest denomination banknotes (less durability with high demand)
Number of denominations	– difficult to calculate and sort – high fixed costs related, among others, to issuance	– non-efficient payment according to the principle of least effort – high production cost caused by the need to keep a large number of monetary items in circulation

[1] The principle of least effort means to use the minimum number of coins and notes during payments. A detailed definition is presented later.

Source: Pattanarangsun (2011).

Table 2

DF, PP and KPSS tests for the order of integration of individual variables

Variable	Level			Difference			Comments
	ADF	PP	KPSS	ADF	PP	KPSS	
10 and 20	2.02319	1.581230	0.896223***	-1.155048	-12.00173***	0.219620	none intercept
200	2.61469	3.322554	0.954028***	-2.795989***	-7.578920***	0.453388*	none intercept
GDP	2.62377	5.085488	1.168494***	-0.903737***	-11.70863***	0.076114	none intercept
CPI	-2.61663* -2.59114	-3.213936** -2.825544	0.483203**	-4.804592***	-4.853145***	0.214659	intercept trend and intercept
WIBOR	-3.69966**	-2.693215* -2.148477	0.834719***	-3.828770***	-3.944632***	0.267970	intercept trend and intercept

Note: the symbols *, ** and *** imply rejection of the null hypothesis at the 10%, 5% and 1% level, respectively.

Source: calculations based on NBP data.

Table 3
Results of the estimation of the long-term relation for demand for 200 zloty and 10–20 zloty notes. Quarterly data for the period 2000 Q4 – 2015 Q4

	Cointegrating equations	
	200 zloty note	**10 and 20 zloty notes**
LOG(notes)	1.000000	1.000000
LOG(GDP)	-1.341587 (0.11969) [-11.2091]	-0.532738 (0.19153) [-2.78144]
LOG(1 + CPI)	10.38436 (1.17357) [8.84853]	11.72863 (2.50216) [4.68740]
LOG(1 + WIBOR)	-3.319917 (0.85451) [-3.88517]	–
C	6.925620	-1.273145
Error correction term	-0.169729 (0.02265) [-7.49265]	-0.021722 (0.00772) [-2.81200]

Notes:
Sample (adjusted): 2000 Q4 – 2015 Q4; standard errors in ( ), t-statistics in [ ].
Due to negative values of inflation since the 3rd quarter of 2014 and the adoption of the log-log model, 1 + WIBOR 3M and 1 + CPI are considered.

Source: analysis in Eviews.

Table 4
Estimation of Laurent's model for 10–50 zloty notes

Variable	**Coefficient**	**Standard error**	**t-Statistic**	**Probability**
C	-0.6456	0.717526	-0.899822	0.3846
LOG(RD)	0.918540	0.230848	3.978986	0.0016
R-squared	0.549117	Mean dependent variable		2.144400
Adjusted R-squared	0.514434	S.D. of dependent variable		0.846006
S.E. of regression	0.589519	Akaike info criterion		1.904546
Sum squared residual	4.517921	Schwarz criterion		1.998952
Log likelihood	-12.28409	Hannan-Quinn criterion		1.903540
F-statistic	15.83233	Durbin-Watson statistic		1.667456
Probability (F-statistic)	0.001573			

Note: dependent variable: AL; method: least squares; included observations: 15.

Source: calculation based on NBP data.

Table 5

Results of the estimation of the hoarding volume of 100 and 200 zloty banknotes as a percentage of these denominations in circulation (in %)

	200			100		
	RFR	**denomination structure of inflow and circulation**	**lifespan**	**RFR**	**denomination structure of inflow and circulation**	**lifespan**
2011	77.75	82.26	75.55	47.41	58.06	59.51
2012	75.63	80.86	75.14	44.77	56.61	63.38
2013	75.41	80.27	74.20	46.25	56.88	68.89
2014	73.27	78.62	72.73	45.75	56.61	74.57
2015	68.56	74.45	70.56	46.88	56.82	79.50

Source: calculations based on NBP data.

Table 6

Results of the estimation of the hoarding value of 100 and 200 zloty banknotes (in PLN million)

	200			100		
	RFR	**denomination structure of inflow and circulation**	**lifespan**	**RFR**	**denomination structure of inflow and circulation**	**lifespan**
2011	16,867.4	17,845.1	16,391.3	30,714.3	37,614.7	38,552.4
2012	16,957.2	18,129.0	16,847.9	30,887.7	39,060.3	43,726.5
2013	18,094.5	19,261.5	17,804.0	33,692.6	41,438.6	50,190.7
2014	19,356.7	20,771.5	19,214.2	35,741.2	44,233.3	58,258.0
2015	20,419.0	22,170.5	21,013.9	40,407.0	48,972.9	68,519.8

Source: calculations based on NBP data.

Table 7
Data for Warsaw based on Union Bank of Switzerland's reports

	2000	2003	2006	2009	2012	2015
USD/PLN exchange rate	0.22856	0.315	0.313	0.283	0.305	0.269
Working days	250	251	251	253	252	252
Working hours	1,870	1,901	1,772	1,756	1,793	1,757
Net hourly wages in USD	1.4	2.2	2.9	4.1	5.5	6.07
Net yearly wages in USD	2,618	4,182.2	5,138.8	7,199.6	9,861.5	10,657.98
Net yearly wages in PLN	11,454.32	13,276.83	16,417.89	25,440.28	32,332.79	39,620.73

Source: calculation based on data from *Prices and Earnings Around the Global*, Reports of the Union Bank of Switzerland (2000, 2003, 2006, 2009, 2015).

Table 8
Wages calculations for Poland

	2000	2003	2006	2009	2012	2015
Ratio of wages in Warsaw to wages all over Poland	1.54	1.47	1.43	1.43	1.44	1.36
Net hourly wages in USD	0.91	1.50	2.02	2.87	3.81	4.45
Net yearly wages in USD	1,698.09	2,853.11	3,582.88	5,041.77	6,839.73	7,818.77
Net yearly wages in PLN	7,429.52	9,057.504	11,446.92	17,815.43	22,425.35	29,066.06

Source: calculations based on GUS data.

Table 9
Characteristics of the D-Metric model for Poland

	2000	2003	2006	2009	2012	2015
D	29.72	36.09	45.61	70.42	88.99	115.34
D/50	0.59	0.72	0.91	1.41	1.78	2.31
D/20	1.49	1.80	2.28	3.52	4.45	5.77
D/5000	0.01	0.01	0.01	0.01	0.02	0.023
5D	148.59	180.43	228.03	352.08	444.95	576.71

Source: calculations based on data from Tables 7 and 8.

Table 10
Denomination structure in 2000 and 2015 according to the D-Metric model

	2000		2015	
	bounds	**denominations**	**bounds**	**denominations**
		coins		
D/5000	0.006		0.023	
		1 grosz		5 grosz
D/2000	0.014		0.06	
		2 grosz		10 grosz
D/1000	0.03		0.12	
		5 grosz		20 grosz
D/500	0.06		0.23	
		10 grosz		50 grosz
D/200	0.15		0.58	
		20 grosz		1 zloty
D/100	0.30		1.15	
		50 grosz		2 zloty
		note-coin boundary		
D/50	0.59		2.31	
		1 zloty		5 zloty
D/20	1.49		5.77	
		notes		
		2 zloty		10 zloty
D/10	2.97		11.53	
		5 zloty		20 zloty
D/5	5.94		23.07	
		10 zloty		50 zloty
D/2	14.86		57.67	
		20 zloty		100 zloty
D	29.72		115.34	
		50 zloty		200 zloty
2D	59.44		230.68	
		100 zloty		500 zloty
5D	148.59		576.71	

Source: analysis based on data from Table 9.

Table 11
Value of banknote denominations in PLN according to PPP for 2014

	Numbers of denominations (1 means the highest denomination)						
Currency	1	2	3	4	5	6	7
CHF	1,160	232	116	58	23	12	–
EUR	1,083	433	217	108	43	22	11
CZK	673	269	135	67	27	13	7
RON	516	207	103	52	10	5	1
PLN with 500	500	200	100	50	20	10	–
HUF	268	134	67	27	13	7	–
BGN	262	131	52	26	13	5	3
PLN	200	100	50	20	10	–	–
SEK	178	89	17.81	9	4	–	–
NOR	167	83	33	17	8	–	–
GBP	112	45	22	11	–	–	1

Source: analysis based on data from websites of central banks.

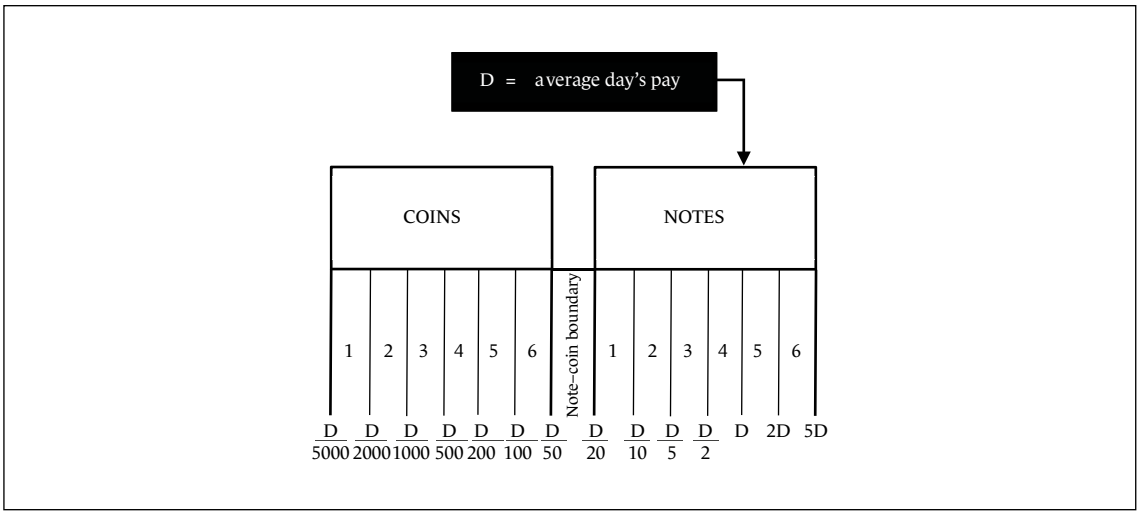
Table 12
Findings of the currency denomination structure analysis in selected European countries and in Israel

	United Kingdom		**Poland**		**Bulgaria**		**Hungary**		**Romania**		**Czech Republic**		**Israel**	
							D =							
	66.29 GBP		**115.34 PLN**		**31.41 BGN**		**6673.66 HUF**		**82.57 RON**		**719.34 CZK**		**299.19 ILS**	
	B	**I**	**B**	**I**	**B**	**I**	**B**	**I**	**B**	**I**	**B**	**I**	**B**	**I**
							coins							
out of CIC		***0.01***		***0.01–0.02***						***0.01***				
D/5000	*0.01*		*0.02*		*0.01*		*1.33*		*0.02*		*0.14*		*0.06*	
1st coin		***0.02***		***0.05***		***0.01***		–		–		–		–
D/2000	*0.03*		*0.06*		*0.02*		*3.34*		*0.04*		*0.36*		*0.15*	
2nd coin		***0.05***		***0.1***		***0.02***		***5***		***0.05***		–		–
D/1000	*0.07*		*0.12*		*0.03*		*6.67*		*0.08*		*0.72*		*0.30*	
3rd coin		***0.1***		***0.2***		***0.05***		***10***		***0.1***		***1***		***0.5***
D/500	*0.13*		*0.23*		*0.06*		*13.35*		*0.17*		*1.44*		*0.60*	
4th coin		***0.2***		***0.5***		***0.1***		***20***		–		***2***		***1***
D/200	*0.33*		*0.58*		*0.16*		*33.37*		*0.41*		*3.60*		*1.50*	
5th coin		***0.5***		***1***		***0.2***		***50***		***0.5***		***5***		***2***
D/100	*0.66*		*1.15*		*0.31*		*66.74*		*0.83*		*7.19*		*2.99*	
6th coin		***1***		***2***		***0.5***		***100***		***1***		***10***		***5***
							note-coin boundary							
D/50	*1.33*		*2.31*		*0.63*		*133.47*		*1.65*		*14.39*		*5.98*	
coin note		***2***		***5***		***1/1***		***200***		***–***		***20***		***10***
D/20	3.31		5.77		1.57		333.68		4.13		35.97		14.96	
							notes							
6th note		**5**		**10**		**2**		**500**		**5**		**50/*50***		**20**
D/10	6.63		11.53		3.14		667.37		8.26		71.93		29.92	
5th note		**10**		**20**		**5**		**1,000**		**10**		**100**		**50**
D/5	13.26		23.07		6.28		1,334.73		16.51		143.87		59.84	
4th note		**20**		**50**		**10**		**2,000**		–		**200**		**100**
D/2	33.15		57.67		15.71		3,336.83		41.28		359.67		149.60	
3rd note		**50**		**100**		**20**		**5,000**		**50**		**500**		**200**
D	66.29		115.34		31.41		6,673.66		82.57		719.34		299.19	
2nd note		–		**200**		**50**		**10,000**		**100**		**1,000**		–
2D	132.59		230.68		62.82		13,347.33		165.13		1,438.67		598.38	
1st note		–		–		**100**		**20,000**		**200**		**2,000**		–
5D	331.47		576.71		157.06		33,368.32		412.83		3,596.68		1,495.96	
out of CIC										**500**		**5,000**		

Notes:
B – bounds, I – items.
Values relating to coins are marked in Italics.

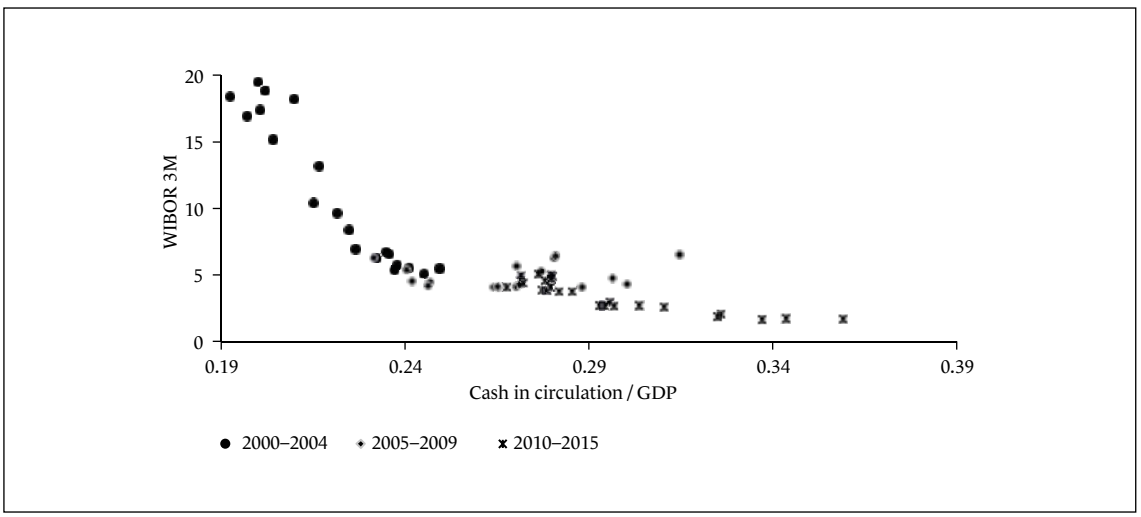
Source: analysis based on *Prices and Earnings Around the Global*, Reports of the Union Bank of Switzerland (2015).

Figure 1
The idea of the D-Metric model



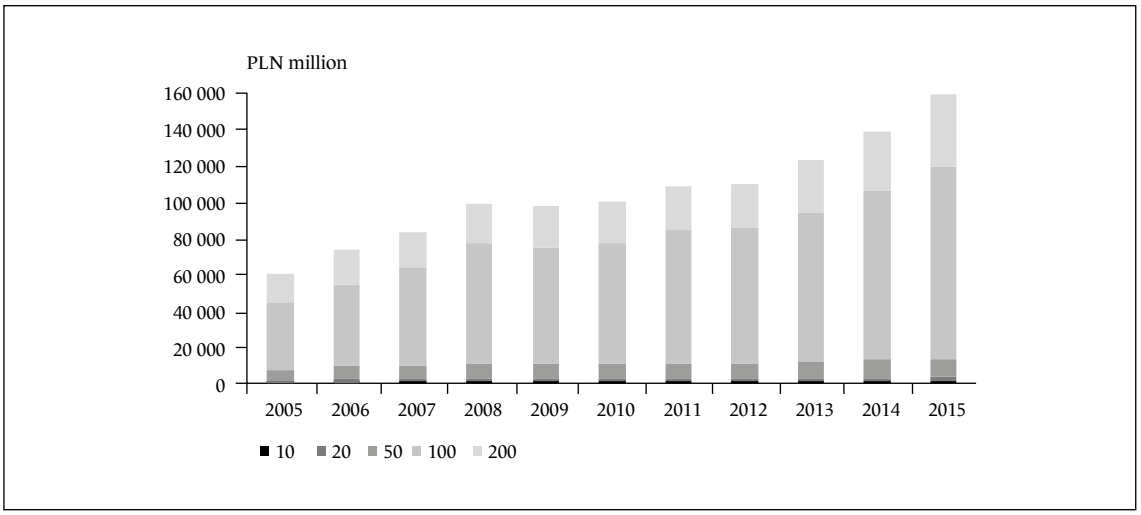
Source: Payne, Morgan (1981).

Figure 2
Currency in circulation in Poland to GDP ratio vs. WIBOR 3M – quarterly data from 2000–2015



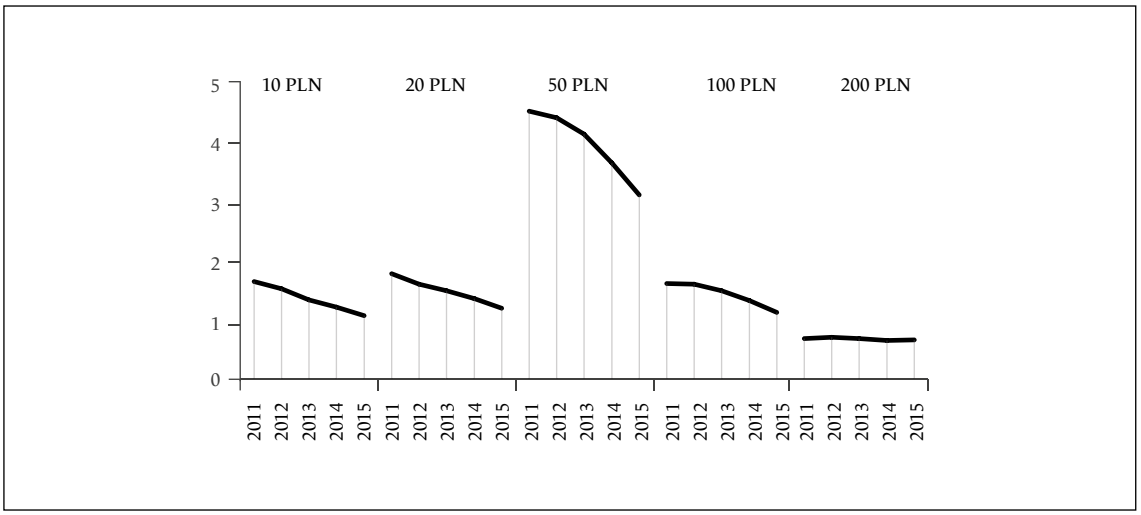
Source: analysis based on NBP data.

Figure 3
Share of notes of particular denominations in circulation in 2005–2015 in Poland



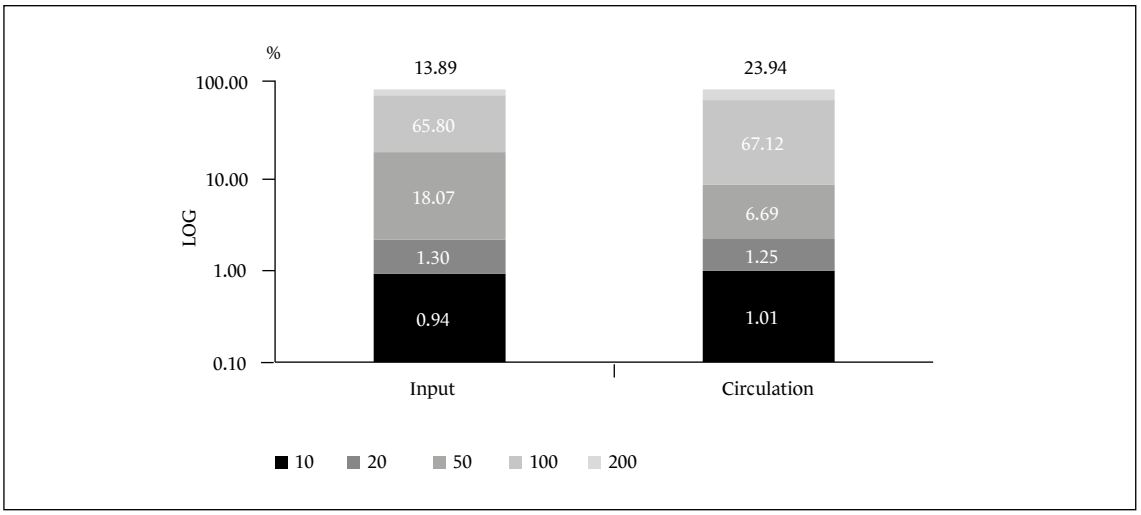
Source: calculations based on NBP data.

Figure 4
Return rates of particular banknotes, 2011–2015



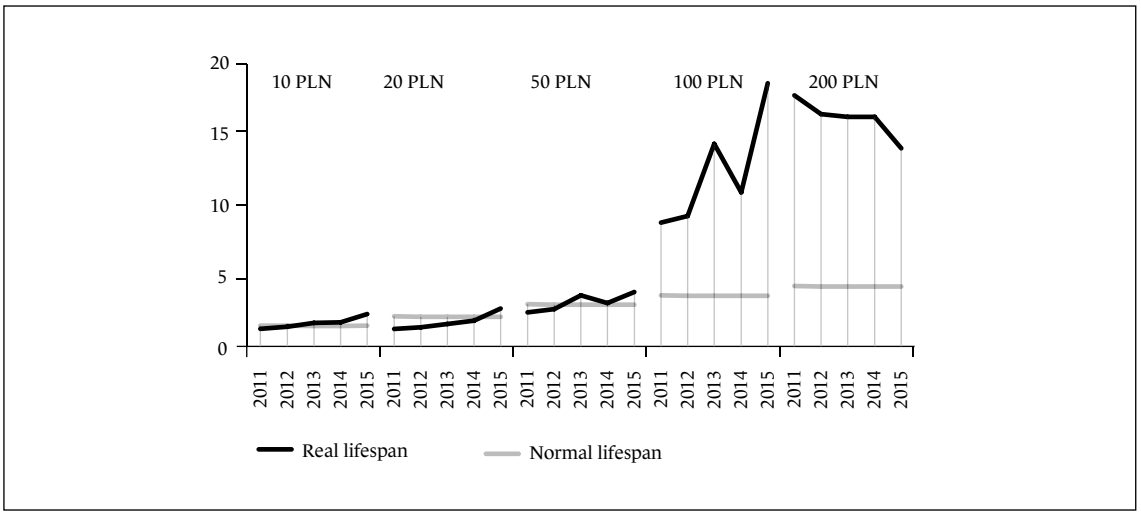
Source: study based on NBP data.

Figure 5
Denomination structure of banknotes in circulation and in input cash flow (on a logarithmic scale)



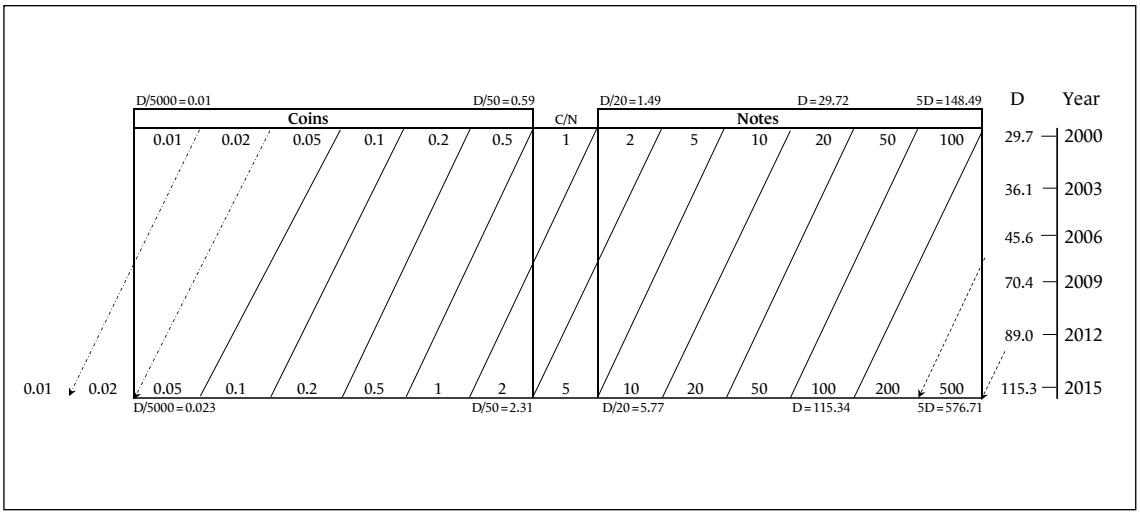
Source: analysis based on NBP data.

Figure 6
Real and normal lifespan of banknotes



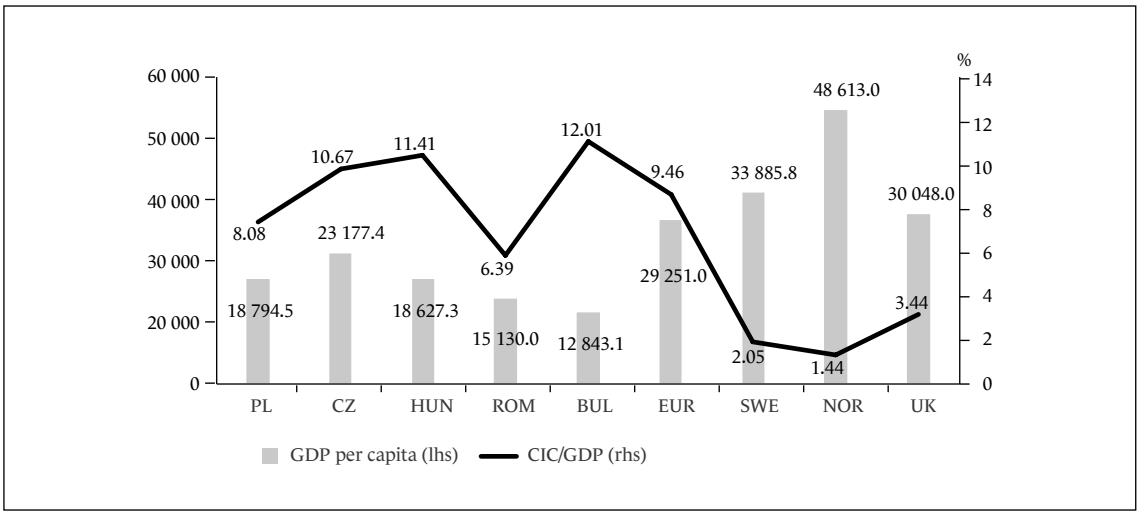
Source: study based on NBP data.

Figure 7
Denomination structure according to the D-Metric model, 2000–2015



Source: analysis based on data from Table 9.

Figure 8
GDP per capita and CIC/GDP ratio in 2014



Source: analysis based on Eurostat data.