Impact of labour market shocks on business cycle fluctuations in Poland

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Abstract
This paper investigates whether labour market shocks contribute to business cycle fluctuations in Poland. To assess the impact of labour market disturbances, we develop an international real business cycle model with non-Walrasian labour market in the spirit of Diamond, Mortensen and Pissarides. To fit the model to the data, we use Bayesian techniques to estimate selected model parameters and shock properties. We find that hiring cost and job destruction rate disturbances were non-negligible factors affecting both output and consumption variability in the period of 1995 Q1 – 2013 Q3. Shock to the workers' bargaining power turns out to have almost no impact on evolution of these two macro variables. Moreover, our results show the condition derived by Hosios (1990) is nearly satisfied, i.e. the degree of inefficiency resulting from the search process decentralization is very small.

Keywords: labour market, search and matching frictions, business cycle

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1. Introduction

In recent years, the economists have increasingly used search and matching theory to model labour markets. Macroeconomic models enriched with search mechanism proved helpful in addressing many issues which simple frictionless models have problems with. They showed that labour market frictions do matter and job creation requires time and effort from both workers and employers.

This paper draws on a vast literature on search and matching theory and focuses on the Polish economy. The analysis is particularly interesting as Poland differs clearly from other countries on which similar studies have been already conducted. The great majority of papers investigating the labour market imperfections focuses on the well-developed, closed economies such as the U.S. or the Eurozone. As Polish economy is relatively small and its trade linkages to other countries are rather significant, the developed model has to account for the effects of foreign economies. Moreover, Poland belongs to the group of transition economies and as a result, the labour market might function differently than in the highly developed market economies.

The contribution of this paper is twofold. Firstly, it develops a general equilibrium model that embeds search and matching frictions in a small economy framework. Secondly, it investigates the importance of labour market dynamics for the Polish economy. Toward this aim, we employ the Bayesian approach to estimate selected parameters and shock properties using the data from the period of 1995 Q1 – 2013 Q3. The set of considered stochastic disturbances includes standard shocks used in small open economy models (one supply shock affecting productivity, two demand shocks to households’ preferences and to government spending, and shock to foreign output) and three labour market shocks chosen in line with Christoffel, Kuester and Linzert (2009) and disturbing separation rate, hiring cost and workers’ bargaining power.

The foundations of search theory used to describe the labour market in our model were laid by Diamond, Mortensen and Pissarides, henceforth DMP, who were the first to introduce search frictions into formal models. In the 1990s Merz (1995) and Andolfatto (1996) took the first attempts to integrate the DMP framework into core real business cycle models. Later on, this approach to modelling labour markets was also incorporated into new-Keynesian models to investigate the effects of labour market imperfections on inflation dynamics and monetary policy transmission (Blanchard, Gali 2010; Christoffel, Kuester, Linzert 2009; Thomas, Zanetti 2009; Trigari 2006). To improve the empirical performance of the standard DMP framework and solve the problem of too high variability of wages and too low volatility of unemployment over the business cycle, several forms of wage rigidities including an adaptive wage rule (Hall 2005) or Calvo scheme (see e.g. Christoffel, Kuester, Linzert 2009) were proposed in the literature.

So far, the models with search frictions have been calibrated or estimated mainly using data from the U.S. (Christiano, Eichenbaum, Trabandt 2013; Christoffel, Kuester 2008; Gertler, Trigari 2009), the euro area (Christoffel, Kuester, Linzert 2009; Thomas, Zanetti 2009) or the U.K. (Faccini, Millard, Zanetti 2013). For instance, Christoffel, Kuester and Linzert (2009) prove that labour market shocks are important determinants of business cycle fluctuations in the Eurozone. However, research considering the effects of a similar set of labour market disturbances on the Polish economy has not been conducted yet, which provides the main motivation for this study.

The remainder of the paper is organized as follows: Section 2 describes the model, Section 3 provides a description of data used in model estimation, Section 4 presents the rationale for calibrated
parameters’ values, Section 5 justifies a choice of prior densities for estimated parameters and discusses estimation results, Section 6 presents the implications generated by the model, Section 7 concludes.

2. Model structure

2.1. Households

Each household consists of a measure of $N_i$ employed members and $U_i$ unemployed members. Following Merz (1995) and Andolfatto (1996), we assume that the family members perfectly insure each other against consumption fluctuations. A problem of the representative household is to maximize its lifetime utility:

$$E^0 \sum_{t=0}^{\infty} \beta^t \varepsilon_{t, \beta} u(C_t, N_t) = E^0 \sum_{t=0}^{\infty} \beta^t \varepsilon_{t, \beta} \left( \frac{C_t^{1-\xi}}{1-\xi} - \kappa^t N_t^{1+\phi} \right)$$

(1)

where:

- $E_t$ - expectation operator taken at time $t$,
- $\beta$ - discount factor describing households’ time preference,
- $C_t$ - composite consumption good,
- $\kappa^t > 0$ - parameter scaling the disutility of work,
- $\xi > 0$ - inverse of intertemporal substitution elasticity,
- $\phi > 0$ - inverse of the Frisch intertemporal elasticity substitution in labour supply.

Additionally, equation (1), similarly to Smets and Wouters (2003) and Mandelman and Zanetti (2008), contains a shock to the discount rate $\varepsilon_{t, \beta}$, which affects the intertemporal rate of substitution between consumption from different periods. This shock is assumed to follow a first-order autoregressive process taking the following form:

$$\ln(\varepsilon_{t, \beta}) = \rho_{\beta} \ln(\varepsilon_{t-1, \beta}) + e_{t, \beta} \text{, } \rho_{\beta} \in [0,1], \text{ } e_{t, \beta} \overset{iid}{\sim} N(0, \sigma^2_{\beta})$$

(2)

Households maximize their lifetime utility facing the law of motion of the physical capital defined by $K_{t+1} = (1-\delta)K_t + I_t$, where $\delta$ denotes the depreciation rate of capital, and the budget constraint. They pool their income from their working members $W_t, N_t$ ($W_t$ denotes a nominal wage), renting physical capital $R_t, K_t$ ($R_t$ denotes a nominal rental rate of capital), benefits received by unemployed households’ members $P_t b U_t$ ($P_t$ is a consumer price index and $b$ denotes real unemployment benefits) and profits from the firms owned by the households $\Pi_t$.

Households spend their income on consumption $P_t C_t$, investment in physical capital $P_t I_t$ and lump-sum taxes levied by the government $T_t$. Moreover, they have access to a full set of Arrow-Debreu securities which implies that the markets are complete.

The household’s optimization problem can be stated as follows:

$$\max_{C_t, K_t, I_t, \beta_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t \varepsilon_{t, \beta} \left( \frac{C_t^{1-\xi}}{1-\xi} - \kappa^t N_t^{1+\phi} \right)$$

(3)
subject to:

\[ P_t C_t + P_t^f I_t + T_t + E_t (Q_{t+1}, D_{t+1}) = P_t b U_t + W_t N_t + R_t K_t + \Pi_t + D_t \]  

(4)

\[ K_{t+1} = (1 - \delta) K_t + I_t \]  

(5)

where \( D_t \) is a portfolio of Arrow-Debreu securities and \( Q_{t+1} \) is a stochastic discount factor.

By substituting (5) into (4) and letting \( \lambda_t / P_t \) denote the Lagrange multiplier on the budget constraint, we obtain first-order-conditions, which, after transformation, give us an Euler equation for consumption:

\[ 1 = E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \frac{1}{P_t} (R_{t+1} + (1 - \delta) P_{t+1}^f) \right] \]  

(6)

where:

\[ \lambda_t = e_{\beta_t} u_C (C_t, N_t) = e_{\beta_t} C_t^{-\gamma} \]  

(7)

Taking into account that we model an open economy, we need to define, how households make a choice between consumption of domestic and imported goods. We assume that consumption is Cobb-Douglas aggregated. The optimal consumption basket composition can be obtained by solving the below optimization problem:

\[ \max_{C_t, C_{H,t}, C_{F,t}} P_t C_t - P_{H,t} C_{H,t} - P_{F,t} C_{F,t} \]  

(8)

subject to:

\[ C_t = \frac{C_{H,t}^{\alpha'} C_{F,t}^{1 - \alpha'}}{\alpha' (1 - \alpha')^{1 - \alpha'}} \]  

(9)

where:

- \( P_{H,t} \) - price of domestic goods,
- \( P_{F,t} \) - price of imported goods,
- \( C_{H,t} \) - domestic consumption goods,
- \( C_{F,t} \) - imported consumption goods,
- \( \alpha' \) - share of consumption allocated to domestic goods.

The optimal allocation of total consumption expenditures yields the demand functions for domestic and imported consumption goods:

\[ C_{H,t} = \frac{\alpha' P_t C_t}{P_{H,t}}, \quad C_{F,t} = \frac{(1 - \alpha') P_t C_t}{P_{F,t}} \]  

(10)

The consumer price index \( P_t \) can be expressed as follows:
\[ P_t = P_{H,t}^{\alpha_1} P_{F,t}^{1-\alpha_1} \] (11)

Similarly, the composite investment good \( I_t \) consists of both domestic and foreign bundles. In order to find demand functions for these two categories of goods, we need to solve the following maximization problem:

\[
\max_{I_{H,t},I_{F,t}} P_t I_t - P_{H,t} I_{H,t} - P_{F,t} I_{F,t}
\] (12)

subject to:

\[
I_t = \frac{I_{H,t}^{\alpha_1} I_{F,t}^{1-\alpha_1}}{\alpha_1 (1 - \alpha_1)^{1-\alpha_1}}
\] (13)

where \( I_{H,t} \) denotes domestic investment goods, \( I_{F,t} \) – imported investment goods and \( \alpha_1 \) corresponds to the share of domestic goods in total investment.

The obtained demand functions are given by:

\[
I_{H,t} = \frac{\alpha_1 P_t I_t}{P_{H,t}}, \quad I_{F,t} = \frac{(1-\alpha_1) P_t I_t}{P_{F,t}}
\] (14)

while the aggregate investment price level takes the following form:

\[
P_t = P_{H,t}^{\alpha_1} P_{F,t}^{1-\alpha_1}
\] (15)

### 2.2. Labour market

The labour market in the model is described similarly to Christoffel, Kuester and Linzert (2009), Gertler and Trigari (2009) and Gradzewicz (2009). The imperfections on this market result from the search and matching frictions.

Household members are employed by intermediate firms. In order to increase employment, a given firm has to actively look for an adequate worker and post a vacancy, which is both time-consuming and costly. The process through which the vacancies and the unemployed workers match with each other is described with the use of standard, well-behaved matching function:

\[
M_t = \sigma^m U_t^\alpha V_t^{1-\alpha}
\] (16)

where:

- \( M_t \) – number of new matches,
- \( \sigma^m \) – parameter describing the matching efficiency,
- \( V_t \) – number of job vacancies.

An average probability that an unemployed worker finds a job is specified as follows:
Employment in the model evolves according to the following law of motion:

\[ N_t = (1 - \rho_t) N_{t-1} + M_{t-1} \]  

where \( \rho_t \in [0,1] \) is an exogenous job destruction (or separation) rate.

Let us note that we assume that new matches become productive for the first time in the next period. This approach is commonly used in the literature (see e.g. Christoffel, Kuester, Linzert 2009; Gertler, Trigari 2009).

In many empirical studies, the job separations are assumed to occur at the constant rate (see e.g. Christoffel, Kuester, Linzert 2006; Gradzewicz 2009; Trigari 2006). The assumption of a constant separation rate seems to be plausible and empirically justified as the studies of Hall (2005) and Shimer (2005) show that during recession the firms are not more willing to reduce their workforce than in other times and there is rather low variation in job destruction rate over the business cycle. According to them, higher unemployment rate, which is observed during the periods of low economic activity, results rather from the employers’ reluctance to hire new workers, and not from the more frequent termination of employment. However, the shock in the job destruction rate proves itself to be an important source of the fluctuations on the labour market in the euro area in Christoffel, Kuester and Linzert (2009). Therefore, following their results, we decide to incorporate the separation shock into our model and assume that the job destruction rate evolves as follows:

\[
\ln(\rho_t) = (1 - \rho_p) \ln(\rho) + \rho_p \ln(\rho_{t-1}) + e_{\rho,t}^i, \quad \rho_p \in [0,1], \quad e_{\rho,t}^i \sim N(0, \sigma_{\rho}^2)
\]  

where \( \rho \) represents the steady state job destruction rate.
2.3. Firms

In our economy there are two firm sectors: perfectly competitive final good sector and intermediate goods sector, which is characterized by monopolistic competition.

Final good producers

On the domestic market, there is a continuum of intermediate firms indexed by $i$ on the unit interval. These firms produce goods $Y_i$ which are, in turn, used in the production process of final good producers. The final good firms combine differentiated $Y_i$ into a homogeneous good using a Dixit-Stiglitz technology:

$$Y_f = \left( \int_0^1 Y_i(p_i) \frac{1}{p_i} di \right)^\mu$$

(23)

where $\mu > 1$ denotes the intermediate producers' gross price markup.

The aim of a representative firm is to choose inputs $Y_i$ and a level of own production $Y_f$ which maximize its profits. Formally, its optimization problem can be expressed as:

$$\max_{Y_f, Y_i} \quad P_{H,t} Y_f - \int_0^1 P_{H,t}(i) Y_i(i) di$$

(24)

subject to the production technology (23).

Solving the problem above yields the demand function for a single intermediate good $Y_i$:

$$Y_i = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{\mu - 1} Y_f$$

(25)

A perfectly competitive final good market drives the firm' profits to zero. The aggregate domestic price level satisfying zero-profit condition is given by:

$$P_{H,t} = \left( \int_0^1 P_{H,t}(i) \frac{1}{\mu} \frac{1}{1+\mu} \right)^{1-\mu}$$

(26)

Intermediate goods producers

Firms in the intermediate sector produce goods according to the constant return to scale Cobb-Douglas production function:

$$Y_i = f(Z_i, K_i, N_i) = Z_i K_i(\alpha) N_i(1-\alpha)$$

(27)
where \( Z_t \) is a productivity level given exogenously by the stochastic AR(1) process:

\[
\ln(Z_t) = \rho_z \ln(Z_{t-1}) + e_{z,t}, \quad \rho_z \in [0,1], \quad e_{z,t} \sim \text{iid} N(0, \sigma_z^2) \tag{28}
\]

Analogously to (21), we can write an equation describing the evolution of employment of the \( i \)-th intermediate producer as:

\[
N_t(i) = (1 - \rho_y) N_{t-1}(i) + M_{t-1}(i) \tag{29}
\]

where \( M_t(i) = q_t V_t(i) \) and the vacancy filling probability \( q_t \) is defined by (18).

Intermediate producers choose the level of capital \( K_t(i) \), employment \( N_t(i) \), price \( P_{H,t}(i) \) and number of vacancies \( V_t(i) \) to maximize a sum of discounted profits facing the demand constraint, production technology and the law of motion of employment. Formally, the \( i \)-th producer’s maximization problem can be stated as follows:

\[
\max_{K_t(i), N_t(i), P_{H,t}(i), V_t(i)} E_{t} \sum_{t=0}^{\infty} \beta_{0,t} E_t \left[ \frac{1}{P_t} \left( P_{H,t}(i) \left( \frac{P_{H,t}(i)}{P_t} \right)^{-\frac{\mu}{\mu-1}} Y_t - W_t(i) N_t(i) - P_{H,t} \kappa_t V_t(i) - R_t K_t(i) \right) \right] \tag{30}
\]

subject to (25), (27) and (29), where \( \beta_{0,t} \) is an equilibrium stochastic discount factor consistent with households’ preferences and is given by:

\[
\beta_{0,t} = \beta^t E_t \left[ \frac{\lambda_t^{1-t}}{k_t} \right] \tag{31}
\]

By substituting equation (25) into (27) and letting \( MC_t(i)/P_t \) denote the Lagrange multiplier on the production technology and \( v_l^t(i) \) denote the Lagrange multiplier on the employment constraint, the FOCs with respect to \( K_t(i), N_t(i), P_{H,t}(i) \) and \( V_t(i) \) are:

\[
\frac{R_t}{P_t} = \frac{MC_t(i)}{P_t} f_{K_t}(i) \tag{32}
\]

\[
v_l^t(i) = \frac{MC_t(i)}{P_t} f_{N_t}(i) - \frac{W_t(i)}{P_t} + E_t [\beta_{t+1} (1 - \rho_{t+1}) v_l^{t+1}(i)] \tag{33}
\]

\[
P_{H,t}(i) = \mu MC_t(i) \tag{34}
\]

\[
\frac{P_{H,t}}{P_t} \kappa_t = q_t E_t [\beta_{t+1} v_l^{t+1}(i)] \tag{35}
\]

where \( MC_t(i) \) gives the \( i \)-th producer’s nominal marginal cost, \( v_l^t(i) \) gives the current period asset value of a worker to the \( i \)-th firm and \( \kappa_t \) represents real vacancy posting cost.
Similarly to e.g. Christoffel, Kuester and Linzert (2009), we allow the vacancy posting cost to follow AR(1) process:

$$\ln(\kappa^*_t) = (1 - \rho_v)\ln(\kappa^*_t) + \rho_v \ln(\kappa^*_t) + e_{v,t}, \quad \rho_v \in [0,1], \quad e_{v,t} \sim \mathcal{N}(0, \sigma_v^2)$$

(36)

where $\kappa^*_t$ is the steady state real vacancy posting cost.

In order to interpret equation (32), note that the marginal cost is in equilibrium equal to marginal revenue. According to this equation, real marginal cost associated with renting an additional unit of capital $R_t / P_t$ have to be equal to the real marginal benefits given by the product of real marginal revenue $MC_t(\ell_t) / P_t$ and the marginal product of capital.

Equation (33) defines the asset value of the occupied job for the firm. This is given by the worker’s contribution to the current profits $[MC_t(\ell_t) / P_t] f_u(\ell_t)$, minus cost connected with hiring the additional employee $W_t(\ell_t) / P_t$, plus the continuation value. In period $t + 1$, with probability equal to $1 - \rho_{t+1}$ the worker survives with the firm and with probability equal to $\rho_{t+1}$ the match is terminated.

Equation (34) defines the way in which the producers determine their prices. While making their pricing decisions, firms follow the markup rule according to which the price is set as a constant markup $\mu$ multiplied by the nominal marginal cost.

Equation (35) describes vacancy posting decision of the firm. In equilibrium, the real cost of posting vacancy multiplied by the relative price of domestic products is equal to the expected value of the match discounted to period $t$ and multiplied by the probability of filling an open vacancy.

**Efficient Nash wage bargaining**

Within our framework, the real wage setting mechanism is based on the individual bargaining between the worker and the firm known as (generalized) Nash bargaining. The aim of this bargaining game is to determine a way in which the joint surplus resulting from the match is divided between the worker and the firm. Noting that in equilibrium the free entry condition drives the value of a vacant job to zero, the $i$-th firm’s surplus can be simply defined as $v\ell^w_i(i)$. The worker’s surplus from the successful match is given by the difference between expected value of income stream of the employed and unemployed worker denoted by, respectively, $v\ell^w_i(i)$ and $v\ell^u_i(i)$. When the match is formed, the worker has to give up $v\ell^u_i$ for $v\ell^w_i$ and the $i$-th producer gains $v\ell^w_i(i)$.

The employment value for the worker depends on the compensation for work given by the real wage $w_i(t)$, utility loss resulting from working and the continuation value. In period $t + 1$ with probability equal to $1 - \rho_{t+1}$ the worker is still employed and with probability $\rho_{t+1}$ the match is destroyed. Therefore, the value function $v\ell^w_i(i)$ can be expressed as follows:

$$v\ell^w_i(i) = w_i(i) - \kappa^* N_i(i)^{1 + \phi} \frac{1}{1 + \phi} + E_{i+1} \beta_{t+1} [v\ell^w_{t+1}(i) + \rho_{t+1} v\ell^u_{t+1}].$$

(37)
The value of a worker in unemployment can be defined by:

\[ v^U_i = b + E_i \beta_{t+1} [s_i v^{w}_{t+1} (i) + (1-s_i) v^U_{t+1}] \] (38)

In the current period, an unemployed receives an unemployment benefit \( b \) and has a chance equal to \( s_i \) of finding a job the next period. With probability equal to \( 1-s_i \) the worker stays unemployed.

The negotiated wage depends on the labour’s bargaining strength of both bargaining participants and satisfies (Christoffel, Kuester, Linzert 2009; Pissarides 2000):

\[ w^N_i (i) = \arg \max (v^U_i (i) - v^L_i (i))^\eta_i \] (39)

The workers’ bargaining power \( \eta_i \) evolves according to:

\[ \ln(\eta_i) = (1-\rho_\eta) \ln(\eta_i) + \rho_\eta \ln(\eta_{i-1}) + e_{\eta_i} \quad \rho_\eta \in [0,1] \quad e_{\eta_i} \sim N(0, \sigma^2_\eta) \] (40)

where \( \eta \) is the steady state workers’ bargaining power.

The first-order-condition of the maximization above takes the following form:

\[ v^U_i (i) - v^L_i (i) = \eta_i (v^L_i (i) + v^w_i (i) - v^L_i (i)) \] (41)

Substituting (33), (37) and (38) to (41) yields the equation for the level of negotiated wage:

\[ w^N_i (i) = (1-\eta_i) \left[ b + \kappa^L \frac{N_i (i)^{1+\phi}}{(1+\phi) \lambda_i} \right] + \eta_i \left[ \frac{MC_i (i)}{P_t} f_{N,i} (i) + s_i E_i \beta_{t+1} v^U_{t+1} (i) \right] \] (42)

Using the definition of the labour market tightness index given by (19) and equation (35), we can transform the above formula into the following form:

\[ w^N_i (i) = (1-\eta_i) \left[ b + \kappa^L \frac{N_i (i)^{1+\phi}}{(1+\phi) \lambda_i} \right] + \eta_i \left[ \frac{MC_i (i)}{P_t} f_{N,i} (i) + \frac{P_{H,t}}{P_t} \kappa^H \Theta_i \right] \] (43)

Thus, the wage rises with the level of employment, marginal costs, market tightness and hiring costs.

**Real wage rigidity**

The empirical literature shows that models in which the wage rate is reset in each period are not able to reflect the volatility of labour market variables accurately. They generate too strong response of wage to productivity shifts, which is not observed in the data. Too large movements in wages lead in turn to the counterfactual estimation of the firms’ profits, employment and unemployment responses (Blanchard, Gali 2010; Gradzewicz 2009; Shimer 2005). In order to match movements in wage and unemployment, many empirical studies use different forms of wage rigidities.
According to Hall (2005), the presence of a surplus resulting from the successful job creation implies that many wage levels are efficient and the wage set in Nash bargaining is only one out of many possible solutions. More specifically, any wage path which generates positive surplus for both parties involved in the bargaining process in all time periods is consistent with the equilibrium. One of the equilibrium wage setting rules proposed by Hall is an adaptive wage, according to which the current possible solutions. More specifically, any wage path which generates positive surplus for both parties implies that many wage levels are efficient and the wage set in Nash bargaining is only one out of many.

where \( w_i = \alpha_w w_{i}^{N} + (1 - \alpha_w)w_{i-1} \) is smoothing parameter.

In our model we use the concept above to determine the equilibrium real wage.

### 2.4. Government

Within our framework, government levies lump-sum taxes \( T \) on households and spends them on government consumption \( g \), and unemployment benefits \( b \). Moreover, we make an assumption that government consumes only domestic goods, which implies that the price of government consumption is given by \( P_{H,t} \). We also assume that government cannot run a deficit. Therefore, the government budget constraint satisfies:

\[
P_{H,t}g_t + bP_U = T_t
\]

Finally we assume that real government consumption is given by exogenous AR(1) process:

\[
\ln(g_t) = (1 - \rho_g)\ln(g) + \rho_g \ln(g_{t-1}) + e_{g,t}, \quad \rho_g \in [0,1], \quad e_{g,t} \sim i.i.d N(0, \sigma_g^2)
\]

where \( g \) is a steady state level of government consumption.

### 2.5. Foreign economy, exchange rate and exports

In our model we make an assumption that the domestic economy is “small” compared to the foreign one. Thus, anything what happens abroad is completely unaffected by what happens in the domestic economy. Therefore, foreign output \( Y^*_f \) is exogenous and follows the AR(1) process:

\[
\ln\left( \frac{Y^*_f}{Y} \right) = \rho_y \ln\left( \frac{Y^*_f}{Y^*} \right) + e_{y,t}, \quad \rho_y \in [0,1], \quad e_{y,t} \sim i.i.d N(0, \sigma_y^2)
\]

where \( Y \) is a steady state value of foreign GDP.
Moreover, we assume that the law of one price holds at all times, which, given that the problems of all agents are symmetric and all firms choose the same price, implies that:

\[ P_{F,t} = e_t P^*_t \]  

where \( e_t \) is the nominal exchange rate and \( P^*_t \) denotes foreign goods price measured in foreign currency.

Following Gali and Monacelli (2005), we also define the bilateral real exchange rate \( Q_t \) as the ratio of foreign and domestic Consumer Price Indices, both measured in domestic currency:

\[ Q_t = \frac{e_t P^*_t}{P_t} \]  

The existence of complete international markets implies that the real exchange rate can be expressed as the ratio of foreign to domestic marginal utility of consumption (Chari, Kehoe, McGrattan 2002). Assuming that the world economy is characterized by the same households' preferences as the domestic one, the international consumption risk sharing condition can be expressed as follows:

\[ \frac{uc^*(C^*_t, N^*_t)}{uc(C_t, N_t)} = \left( \frac{C^*_t}{C_t} \right)^{\epsilon_t} = Q_t \]  

where \( C^*_t \) is foreign consumption.

Finally, we define the total demand by foreigners for domestically produced goods as (Christiano, Trabandt, Walentin 2011):

\[ X_t = \psi \left( \frac{P_{H,t}}{e_t P^*_t} \right)^{\omega} Y^*_t \]  

where \( X_t \) denotes exports, \( \psi > 0 \) is exports scaling parameter and \( \omega > 0 \) represents the relative price elasticity of demand for exports.

### 2.6. Market clearing

Several market clearing conditions must hold in equilibrium. Firstly, the total demand for capital by all intermediate producers has to be equal to its supply offered by households:

\[ K_t = \int_0^1 K_*(i) \, di \]  

Similarly, the aggregate numbers of employed workers and vacancies have to satisfy:

\[ N_t = \int_0^1 N_*(i) \, di \]
\[ V_t = \int_0^1 V_t(i)di \] (54)

Moreover, net foreign assets in equilibrium are given by:

\[ E_t(Q_{t+1}, D_{t+1}) = D_t + NX_t \] (55)

where \( NX_t \) denotes net exports expressed in nominal terms and is defined by:

\[ \text{NX}_t = P_{H,t}X_t - P_{F,t}(C_{F,t} + I_{F,t}) \] (56)

Using equations above and combining budget constraints of all agents of the model yield the standard resource constraint in the final goods market, which is given by:

\[ Y_t = C_{H,t} + I_{H,t} + g_t + \kappa_t'V_t + X_t \] (57)

Finally, we have to ensure that the market of final foreign goods clears. Assuming that there is no distinction between investment and consumption goods abroad, world market clearing condition can be stated as follows:

\[ C_t = Y_t \] (58)

So far all of the model variables have been expressed in nominal terms. As a result, there are many price levels consistent with the equilibrium. However, the relative prices are, up to normalization, unique. That is why, in the next sections we use the model expressed in real terms, i.e. we express all nominal variables relative to \( P_t \). To simplify the notation, we denote nominal variables with capital letters, whereas the variables expressed in real terms are denoted with the corresponding small ones.

### 3. Data

The model is fitted to Polish data using the quarterly data covering the time span of 1995 Q1 – 2013 Q3. The series for output, consumption, government consumption, unemployment, wages per employee and foreign output are employed as observable variables.

As a measure of output we choose real Gross Domestic Product. Real consumption is measured by the final consumption expenditure of households and real government consumption by the final consumption expenditure of general government. All these three variables are taken from the Eurostat, measured at market prices in millions of national currency, chain-linked volumes (reference year 2005) and normalized by the size of population, which is taken from the LFS published by the Central Statistical Office (CSO) in Poland. It has to be noted that a methodological change and a new base for generalization of the LFS results were introduced in the third quarter of 2012. As a consequence, recent LFS results are not fully comparable with the results of surveys preceding this period. The CSO published the corrected data only for the period of 2010 Q1 – 2012 Q2. However, to assure full
comparability of the data, Saczuk (2014) recalculated the results of the Labour Force Surveys from the time span of 1995–2010. In order to express the variable referring to the domestic economy in per capita terms, we divide it by the size of population aged 15 or more, taken from the corrected LFS results.

Time series of the unemployment rate is taken from the study Registered Unemployment published by the CSO in Poland. We choose this data source instead of LFS statistics to assure consistency with number of vacancies which is taken also from this publication. Note that there was a change in a way the unemployment rate is measured in 2002. For this year, the CSO provides the values of unemployment rates calculated in accordance with both old and new methodology. The average difference between the unemployment rates determined with these two approaches amounts to 2 percentage points. In order to assure data consistency, we add 0.02 to the unemployment rates from period of 1995 Q1–2001 Q4. We use the number of job offers submitted to labour offices to measure vacancies and normalize it by the size of population. Average real wage per employee is obtained by dividing the average monthly gross wage and salary in the national economy reported by the CSO in Poland by the GDP deflator.

Real output per capita in the euro area (comprising 18 member states) is measured by Gross Domestic Product at market prices in euro per inhabitant, volume reference and exchange rates 2005, which is taken directly from Eurostat.

All considered variables are seasonally adjusted using the TRAMO SEATS procedure. They are, except the unemployment rate, expressed in logs. All time series are also filtered using Hodric-Prescott filter with $\lambda = 1600$.

4. Model calibration

Due to identification problems, it was impossible to produce the plausible values for all parameters in the estimation procedure. Therefore, we decided to estimate those parameters which are in our opinion most important for the considered problem and fix the rest of them using outside evidence and results presented in other studies. Log-linearized model equations used in estimation are reported in the Appendix.

In order to calibrate the parameters describing the composition of the consumption and investment baskets, we use results reported by Brzoza-Brzezina, Jacquinot and Kolasa (2014), according to which the share of imported consumption goods in total consumption in Poland is equal to 18.43% and the share of imported investment goods in total investment is equal to 65.57%. To satisfy this relations, we set $\alpha^c = 1 - 0.1843 = 0.8157$ and $\alpha^m = 1 - 0.6557 = 0.3443$. The scaling parameter in the households’ utility function is set to $\kappa^L = 1.8969$ in order to match the steady state unemployment rate $U = 0.15$, which is the average over the collected data sample.

The steady state separation rate $\rho$ is calibrated on the basis of results documented in Hobijn and Sahin (2007). According to their estimates based on the data from the period of 1997–2004, the average monthly separation rate in Poland is equal to 0.0099. To convert it into the quarterly terms, we use the following formula: $\rho^q = 1 - ((1 - \rho^m)^3)$, which gives us the quarterly job destruction rate equal to 0.0294. Following Christoffel, Kuester and Linzert (2009), we also analyse the Eurostat data about the share of persons whose job started within the past 3 months in total employment, which can give us the general idea of worker flows on the labour market. The average share of workers who started their jobs
within the last 3 month calculated over the period of 2008 Q1 – 2013 Q4 is equal to 0.0373. However, this approximation of the separation rate usually significantly (even two or three times) overestimates the real values of the job destruction rate (Christoffel, Kuester, Linzert 2009). Taking into account all the above, we decide to set the steady state separation rate to 0.03.

As we do not have any direct evidence about the vacancy filling probability in Poland, we determine it on the basis of the estimate commonly used in the models calibrated both for the U.S. (Cooley, Quadrini 1999; Trigari 2006) and the euro area (Christoffel, Kuester, Linzert 2009) and set \( q = 0.7 \). The parameter governing the matching efficiency \( \sigma_m \) is obtained from the steady state calculations. The probability of finding a job \( s \) implied by the steady state relationships is equal to 0.17, which means that, on average, an unemployed worker finds the job after about 6 quarters.

To calibrate the replacement rate reflecting the outside option of the worker, we use data published by OECD in *Benefits and wages: statistics* study. We take data for Poland from 2012 as it is the most recent statistics available on the OECD website. OECD provides net replacement rates for different family characteristics and income levels. The data used for the calibration of replacement rate in our model is reported in Table 1. Following Christoffel, Kuester and Linzert (2009), we simply calculate the average over all presented categories which equals roughly to 0.4798 and set \( b/w = 0.4798 \).

Lacking the relevant evidence about vacancy posting costs and any estimations of \( \kappa^v \) parameter for the Polish labour market, we decide to calibrate \( \kappa^v \) to meet the steady state hiring costs to output ratio of 0.005 i.e., \( \kappa^v Y/Y = 0.005 \). In the empirical studies for the U.S. economy, this ratio is often set to 0.01 (Blanchard, Gali 2010; Christiano, Eichenbaum, Trabandt 2013). However, in Christoffel, Kuester and Linzert (2009), the ratio obtained for the euro area is equal to 0.0023. Our vacancy posting cost to GDP ratio reconciles both calibration values.

The parameter \( \mu \) is calibrated to 1.1 in order to meet the price markup of 10%. This is the standard value used in the literature (see e.g. Christoffel, Kuester 2008; Gradzewicz 2009). For the households’ time-discount factor \( \beta \) we choose the common value of 0.99 (Smets, Wouters 2003). The quarterly depreciation rate of capital \( \delta \) is set equal to 0.025 implying the annual depreciation of 10% (Smets, Wouters 2003).

The steady state government spending \( g \) targets to satisfy \( g/Y = 0.1823 \), which is the average government spending to output ratio in the collected sample. The steady state investment to output ratio is calibrated to 0.212 based on the work of Brzoza-Brzezina, Jacquinin and Kolasa (2014). The parameter \( \alpha \) in the firms’ production function is set to 0.3274 to meet this relationship. This seems to be a reasonable value as in many empirical studies the elasticity of output with respect to capital is assumed to be equal to around one third.

For simplicity, we make the assumption that in the steady state the net foreign assets \( D \) are equal to zero, which implies that the net exports are also at the zero level. Moreover, we assume that the steady state level of consumption in the domestic economy and abroad are the same. This entails that both the steady state real exchange rate \( Q \) and the relative prices \( p_{F} \) and \( p_{H} \) are equal to unity. Therefore, according to equation (56), net exports are zero when the sum of imported consumption and investment goods is equal to exports. The exports to output ratio implied by these steady state relationships is equal to 0.2497. This seems to be a plausible result taking into account that in our model we assume that the import content of export is equal to 0 (i.e. the exports are produced using only domestic goods), whereas in fact it is equal to around 45% (Brzoza-Brzezina, Jacquinin, Kolasa 2014) and the share of exports in GDP for the Polish economy is currently equal to about 45%.

The rest of the model parameters and shock properties are estimated with Bayesian methods.
5. Prior and posterior distributions

Table 2 presents prior and posterior distributions of the estimated parameters. As for many parameters the relevant studies on Poland are not available, we base the prior selection largely on the studies on the U.S. and countries of Western Europe. The choice of priors is made as follows:

The inverse of the elasticity of intertemporal substitution, $\varsigma$: We assume the risk aversion coefficient to be gamma distributed with prior mean equal to 1.5, which is a common value used in the business cycle literature (see Brzoza-Brzezina, Jacquinot, Kolasa 2014; Christoffel, Kuester, Linzert 2009). We allow for the standard deviation of 0.4.

The inverse of the Frisch elasticity of labour supply, $\phi$: Gamma distribution with mean value of 2 was chosen relying on the earlier studies including Smets and Wouters (2003), Brzoza-Brzezina, Jacquinot and Kolasa (2014) and Christoffel, Kuester and Linzert (2009). Similarly as for $\varsigma$, we impose the standard deviation of 0.4.

The elasticity of matches with respect to unemployment, $\sigma$: The parameter is assumed to follow beta distribution (covering the range between 0 and 1) with a mean equal to 0.6 and a standard deviation of 0.05. The mean value was chosen on the basis of a study of Petrongolo and Pissarides (2001) carried out for the U.S. and European countries, according to which $\sigma$ ranges from 0.5 to 0.7. We select our mean value at the midpoint of these estimates. The same prior distribution for the elasticity of matches with respect to unemployment was used in Christoffel, Kuester and Linzert (2009).

The bargaining power of workers, $\eta$: We assume beta distribution with a mean equal to 0.5, which is a value conventionally used in the search and matching literature and describes symmetric bargaining situations. We allow for the prior standard deviation of 0.05.

The real wage rigidity, $\alpha_w$: To the best of our knowledge, there exists no relevant research that could help to set this parameter for the Polish economy. Therefore, we assume $\alpha_w$ to be beta distributed with the mean of 0.5, which is the midpoint in the range of all values possible for this parameter. We impose the standard deviation of 0.1.

The relative price elasticity of demand for exports, $\omega$: We choose gamma distribution for our prior and assume the proportional movements of the relative price and demand for exports i.e. the unit elasticity of demand. We allow for the wide standard deviation of 0.5.

Serial correlations of shocks: Autoregressive coefficients of all shocks are assumed to follow beta distribution with a mean equal to 0.85 and a standard deviation of 0.1. This is a standard distribution for the parameters describing shock inertia used by e.g. Smets and Wouters (2003).

Standard deviations of shocks: As we want to incorporate only little prior information on the shocks’ volatilities, we choose inverse gamma distributions with infinite standard deviations for the priors. This is a standard approach used in the literature for the modelling of shocks’ standard deviations. As preliminary experimentations with the model have shown that labour market shocks have significantly higher volatilities than other shocks, we impose higher prior means for them. Thus, standard deviations of vacancy posting cost and separation shocks are assumed to follow inverse gamma distribution with the mean of 0.1 and the bargaining power shock with the mean of 0.2. Prior means for the rest of the shocks are set equal to 0.01.

The estimation results reported in Table 2 are obtained with the use of the Metropolis-Hastings algorithm implemented in the Dynare software. On the basis of the Brooks and Gelman diagnostic charts (1998), we assessed the convergence of generated Markov chains and decided to keep last 650,000 out of 1,300,000 draws.
The graphical representation of posterior distributions is shown in Figure 1. On its basis, we can confirm the presumption that the data is rather informative for all parameters as the prior distributions differ remarkably compared to the relevant posterior ones. The only exception here seems to be the elasticity of matches with respect to unemployment, whose prior is not substantially updated with the data. Moreover, it should be mentioned that posterior distributions for all parameters seem to be relatively tight as compared to their prior equivalents.

The point estimates (calculated as means of posterior distributions) of the households’ relative risk aversion $\varsigma$ and the inverse of the Frisch elasticity of labour supply $\phi$ are equal to 1.1722 and 1.6133 respectively. The elasticity of matches with respect to unemployment $\sigma$ is estimated at 0.6241, meaning that a one percent increase in unemployment leads, ceteris paribus, to a 0.6241% rise in the outflow from unemployment. This value is roughly consistent with the estimates of Polish matching functions reported by e.g. Rogut and Tokarski (2002) and Roszkowska (2009), which confirmed that the role of the unemployment in matches formation is bigger than that of the number of vacancies.

The estimated workers’ bargaining power is equal to 0.6167. Interestingly, this value is very close to the estimate of the efficiency of matches with respect to unemployment. Therefore, our estimates nearly satisfy a Hosios (1990) condition, according to which the equalization of the workers’ bargaining power and the share of the unemployed in the matching technology ensures the efficiency of the equilibrium. When the workers’ bargaining power is too high, the firms find it unprofitable to open vacancies. Thus, the number of posted free jobs is small and the unemployment rate increases above the Pareto efficient one. In the opposite case, when the workers’ bargaining power is too low, the employers have a strong incentive to post vacancies and the unemployment reaches the level that is below the one prevailing in the efficient equilibrium (Faia 2009).

The real wage rigidity $\alpha_w$ is estimated at 0.8153, which implies that the current period wage is determined in 81.53% by the solution to the Nash bargaining game and in 18.47% by the wage from the previous period.

The inertia of exogenous disturbances was the strongest in case of the foreign output and vacancy posting cost shocks (AR(1) coefficients of 0.8927 and 0.8826 respectively) and the weakest in case of the shock to the workers’ bargaining power (AR(1) coefficient of 0.2270). The AR(1) coefficient of the job destruction rate shock is estimated at 0.5201. These values are qualitatively similar to the ones obtained by Christoffel, Kuester and Linzert (2009). Their estimation for the euro area reveals that the vacancy cost shock is the most persistent (AR(1) coefficient of 0.78) and the bargaining power shock is characterized by the lowest inertia (AR(1) coefficient of 0.09) among all three considered labour market shocks. The serial correlation of the separation shock obtained by them is equal to 0.51, which is nearly the same as our estimate.

As expected, a shock to the bargaining power turns out to be the most volatile with the standard deviation of 0.3742. The lowest standard deviation is estimated for the preference shock.

6. Model properties

6.1. Comparison with Polish economy

Table 3 delivers a comparison between selected descriptive statistics from the data and those generated by the model. All in all, the model performs well at explaining the relative volatilities. The series which
are less (more) volatile than output in the data are also less (more) volatile in the model. Similarly, signs of the most correlations with output are consistent with the data. In particular, it is true for all labour market variables. The only correlation coefficient whose sign is inappropriate is the one related to government spending. According to the model, government consumption is roughly countercyclical, whereas in fact it is characterized by slight procyclicality.

It is, however, fair to remark that the model has problems with reflecting the strength of relationships between the individual series. In absolute terms, it strongly overestimates standard deviations of both domestic and foreign outputs, consumption and vacancies and underestimates the volatility of the unemployment rate. However, it appears that the model does well at capturing the volatility of government spending (0.0142 versus 0.0135 in the data) and, more interestingly, of the real wage (0.0335 versus 0.0363 in the data).

The strength of the correlation between output and other considered series is too strong in case of consumption and wage and too weak for the rest of variables. Note that the positive correlation between output and vacancies is weakened by the shock to the separation rate as it leads both to lower output and more vacancy posting (see Figure 2). In addition, as many empirical studies (see e.g. Christoffel, Kuester, Linzert 2009), our model fails to capture the Beveridge curve and generates too weak negative correlation between the unemployment rate and vacancies (-0.0681 versus -0.3143). Problems with describing the empirical Beveridge curve are, similarly to the difficulties with matching the correlation between output and vacancies, closely related to the separation shock, which creates counterfactual positive correlation between unemployment and job offers (Shimer 2005). The impulse response plots presented in Figure 2 show that both unemployment rate and the number of vacancies respond positively to the separation shock, which spoils the negative correlation between these two variables. However, it appears to explain the significant share of unemployment volatility (see Table 5) and for this reason we decide to keep it in our model.

Table 4 lists the conditional correlations between unemployment and vacancies, i.e. correlations appearing when only one shock hits the economy and standard deviations of the rest of stochastic disturbances are set to 0. It is clear that only one shock out of three considered labour market disturbances generates correct sign of correlation between unemployment rate and number of vacancies. Reported results confirm that the main force which spoils the Beveridge curve is the separation shock. It creates very strong positive unemployment-vacancies correlation, which is equal to 0.9909. Moreover, it should be mentioned that it is actually the only shock that destroys the negative relationship between these two variables. The positive correlation generated by workers’ bargaining power shock (equal to 0.0423) is rather negligible. On the contrary, hiring cost shock creates relatively strong negative relationship between unemployment and job offers.

6.2. The role of the labour market shocks

Impulse response functions

The analysis of the impulse response functions allows to evaluate dynamic properties of the model. IRFs plots illustrate the deviation of the trajectory of a variable following a shock from its steady state
level. Figure 2 depicts the mean and the 90% uncertainty bands for the impulse responses to shocks equal to one standard deviation.\footnote{The results presented in Figure 2 are based on 1,200 draws from the posterior distribution of the model’s parameters.}

Due to the positive separation shock, more workers lose their jobs and the unemployment rate increases. Moreover, it forces the intermediate firms to make more recruitment effort and post more vacancies in order to bridge the workforce gap caused by the intensified separation. The increase in job offers is, however, stronger than the rise in the unemployment rate, which tightens the labour market. Higher tightness index leads, in turn, to the short-lasting increase in wages. However, after two or three quarters, the effect of the reduced employment starts to dominate and the real wage drops below its steady state level. In addition, lower employment drives down economy’s production capabilities and leads to the output decline. Higher unemployment rate together with lower wage negatively affect also the consumption level. The job finding probability is influenced positively by the increased number of job offers available on the market, which contributes to the gradual reverting of the unemployment rate to its steady state value. As a result, other variables also return to their baseline levels and effects of positive separation shock disappear.

Turning to the hiring cost shock, it is quite obvious that it discourages employers from recruiting activity and decreases the number of available job offers. Less posted vacancies make it harder for a worker to find a job and leads to the unemployment increase. Moreover, positive vacancy posting cost shock translates immediately into lower production and creates the initial peak in the wage’s path. However, as in case of the separation shock, after several periods, the wage falls below its baseline level. Output decline, lower wages and necessity to cover additional costs make that also consumption experiences prolonged negative deviation from its steady state level. Note that effects on output, consumption and wages are quite persistent and do not die out after 10 years.

As far as the positive shock to the workers’ bargaining power is concerned, it should be clear that it exerts upwards pressure on wage and contributes to its increase. Obviously, weakening of the employers’ bargaining position entails the reduction of posted vacancies and leads to the unemployment rise. Similarly to two other considered labour shocks, the disturbance to the bargaining power generates the negative response of both output and consumption. However, the amplitude of their deviations from the steady state levels is much lower.

**Variance decomposition**

Thanks to the incorporation and estimation of several structural shocks, we can investigate the relative contribution of different stochastic disturbances to the overall volatility of the analysed time series. Table 5 delivers the decomposition of effects of various shocks at four different time horizons: upon impact, short run: horizon of 1 year, medium run: horizon of 2.5 years and long run: horizon $\infty$.

The main driving force for output fluctuations turns out to be a technology shock. On impact, it accounts for more than 96% of output variations. Over the longer horizons, its role diminishes (to less than 60% in the long run), but it still appears to be of the major importance for explaining economic activity evolution. The role of the preference and foreign shocks seems to be similar at the 1-year horizon. However, in the medium and long run, the shock in the foreign economy is more important and accounts for, respectively, 10.76% and 19.30% of the forecast error variance. The impact of the
government spending shock is negligible at all forecast horizons. This is the case not only for output fluctuations, but also for the dynamics of the other considered series.

The role of labour market shocks for output fluctuations increases with the forecast horizon from 3.86% to 12.46% in the long run. It is worth noting that the shock to the separation rate is, together with the technology shock, one out of only two shocks affecting output immediately (see impulse responses presented in Figure 2). In the short run, the separation shock turns out to be of the greatest importance among all labour market shocks, with the contribution to the forecast error variance equal to 6.47%. However, its role declines over time, whereas the vacancy posting cost shock becomes more significant determinant of the volatility of the economic activity. As a result, in the long run, the shock to cost of filling the open vacancy is the one that takes the first place among all labour market shocks in explaining output fluctuations in Poland. The workers’ bargaining power shock seems to play no role in explaining output variability. Its contribution to the variance decomposition is the highest at the 4-quarters horizon and is equal to only 0.29%. It should be noted that our conclusions regarding the impact of labour market shocks on the economic activity are different than those made by Christoffel, Kuester and Linzert (2009). Their research conducted for the euro area revealed that bargaining power disturbances carry more information about output evolution than other labour market shocks. However, similarly to our model, the study of Antosiewicz and Lewandowski (2014) investigating the potential sources of macroeconomic fluctuations in eight European countries, including Poland, has shown that job destruction shocks are more important for output dynamics than bargaining power disturbances in all considered countries.

Turning to consumption, we find that there are three structural shocks accounting for the substantial fraction of its variability: the preference shock, the technology shock and the shock to foreign output. The first one plays the dominant role in the short run, the second one in the medium run and the last one is the most important in explaining consumption dynamics in the long run. Relationships between the importance of individual labour market shocks at the particular time horizons are the same as for output. Thus, the separation shock brings most information at the 1-year horizon and the vacancy posting cost shock is the most important among all labour market shocks in the long run. However, quantitatively, effects on consumption volatility are weaker. Moreover, it should be noted that all labour market shocks have an immediate effect on consumption.

Unsurprisingly, within our framework, variations in the unemployment rate, vacancies and real wage are driven primarily by the labour market shocks. Generally, the separation rate and the cost of hiring disturbances bring valuable information about the dynamics of unemployment and vacancies, and not of the real wage, whereas the bargaining power shock accounts for the lion’s share of wage fluctuations, and is only of the minor importance for the unemployment evolution. More precisely, we observe that the separation shock is the key determinant of unemployment dynamics in the short run and the vacancy posting cost plays major role in both medium and long run. The number of open vacancies is very strongly affected by the shock to hiring costs (the forecast error variance of more than 60% at all horizons), but the bargaining power and separation shock also make sizeable contribution to its dynamics: the latter dominates over the former in the short, medium and long run. Other shocks play almost no role for the unemployment and vacancies evolutions. In terms of the wage, the technology shock seems to be the significant determinant of its variability with the forecast error variance of 13.82% in the long run.

All in all, in our model the labour market seems to be only weakly connected to the real sector block. Labour market shocks do not appear to be a significant source of output and consumption variability,
whereas they are responsible for almost entire dynamics of the labour market variables. The research of Antosiewicz and Lewandowski (2014) shows that in case of output this observation is true also in other European countries. Their model identified the foreign demand and productivity shocks as two important determinants of output fluctuations. The role of other disturbances, including bargaining power and job destruction shocks appeared to be of a little help in explaining output volatility. The sources of consumption variability identified by the model of Antosiewicz and Lewandowski were, however, different than those in our model. Contrary to our model, in their framework job destruction shock was an important factor influencing private consumption both in Poland and in six other European countries. The only country in which the effects of job separation disturbances were negligible was the Czech Republic. It should be, however, noted that their model accounts from three labour market states (employment, unemployment and inactivity) and endogenous job destruction rate which might be the reason why the transmission mechanism differs from the one in our model.

**Historical decomposition**

The estimation of a set of stochastic disturbances allows us also to gauge to what extent various shocks explain the past evolution of the given data series. We focus mostly on contribution of different labour market shocks to the movements of output and consumption.

Figure 3 depicts the shock decomposition of selected variables generated by our model along with HP-detrended values of corresponding time series. At the first glance it is clear that output fluctuations are driven mostly by the productivity shocks. However, in line with the variance decomposition results, two labour market shocks also contribute to its evolution. In 1995 and 1996 the positive shock to the job destruction rate drove down output and led to offsetting the positive technology shock. The series of negative labour market shocks leading to the upsurge in the economic activity came in the consecutive periods. It is worth noting that from the second half of 1997 to the third quarter of 1998, separation rate disturbances had the greatest effect on the output movements among all considered structural shocks. In the last quarter of 1998 we could observe the consequences of Russian financial crisis on the Polish economy leading to the significant, but luckily only transient, output decline. In this period labour market shocks helped to countervail very strong negative technology shock, whose effects were additionally enforced by the preference and foreign disturbances. In addition, it has to be mentioned that only between 1998 and 2001 we could observe the sizeable effects of the bargaining power shock on the output dynamics. From 2002 onwards, the role of this disturbance for explaining output variability is hardly noticeable. In period ranging from 2001 to the second half of 2006 Polish economy experienced prolonged slowdown in the economic activity interrupted by quite substantial improvement in 2004, when Poland joined the European Union. In this time span we observed relatively strong positive disturbances to both the separation rate and vacancy posting cost. These two labour market shocks also made a significant contribution to the peak in the economic activity in 2007 and first two quarters of 2008. In 2009 Poland started to experience the consequences of the global recession. It is worth noting that negative hiring cost shocks quite significantly helped to limit the decline in output in this period. From 2011 onwards, we observe the series of positive vacancy posting cost disturbances, which clearly leads to deepening the economic downturn connected to the second wave of Eurozone crisis. All in all, one can notice that during previous economic slowdown lasting form 2001 to 2006 both hiring cost
shock and shock to the job destruction rate affected output dynamics, whereas during current financial crisis, only the first one matters and impact of the latter is rather negligible.

From Figure 3 it is clear that labour market shocks are less important for the evolution of consumption than in case of output. This is particularly true in the most recent periods when they played almost no role for consumption dynamics. However, it should be noted that the hiring cost shock significantly helped to alleviate negative effects of global financial crisis in 2009 and prevented consumption from deviating negatively from its long term trend. Moreover, it has to be pointed out that separation shock quite successfully offset very strong negative preference shock which occurred in consequence of Russian financial crisis in the end of 1998.

Turning to the historical decomposition of unemployment, vacancies and wage, we can see that labour market shocks were of the greatest importance for the dynamics of these three macro variables. Unemployment and vacancies fluctuations were driven predominantly by separation and hiring cost shocks, whereas the workers’ bargaining power disturbances were the driving force for real wage variability.

7. Conclusions

This paper aims to investigate the impact of labour markets on business cycle fluctuations in Poland. We focus mostly on implications for output and consumption dynamics. To assess the role of labour market we construct the small open economy model enriched with search and matching mechanism developed in the spirit of Diamond, Mortensen and Pissarides. Apart from a set of stochastic disturbances which are traditionally used in the small open economy models, our framework includes three labour market shocks: shock to the job destruction rate, to hiring cost and to workers’ bargaining power. To fit the model to the data, part of model parameters is estimated with Bayesian methods.

Our results can be summarized as follows. First, the Hosios rule, which is satisfied when workers’ bargaining power and elasticity of matching function with respect to unemployment are equal, nearly holds in the Polish economy. This suggests that there are almost no inefficiencies resulting from the decentralization of the search and matching process.

Second, labour market shocks play a role for business cycle fluctuations in Poland. The study points towards disturbances in vacancy posting cost and separation rate as non-negligible determinants of output evolution. These two shocks contributed quite significantly to the economic slowdown from the period of 2001–2006. However, during current recession separation shock plays almost no role. Therefore, one can state that the behaviour of labour market during current economic downturn is somewhat different than during the previous one. Vacancy posting cost and separation rate disturbances prove themselves to be the crucial contributors to vacancy and unemployment variability, whereas the bargaining power shock turns out to be a driving force of wage volatility.

In our model we assumed that the separation rate is exogenous. As our analysis has shown that job destruction is a powerful source for data dynamics, one of the interesting extensions to the model would be allowing for endogenous job separation in order to shed additional light on its relevance for aggregate data fluctuations. Estimating a model with endogenous job destruction on the basis of Polish data is therefore an important task for future research.
References


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\section*{Appendix}

Table 1
Net replacement rates in Poland

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<th></th>
<th>Initial</th>
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<tr>
<td></td>
<td>67% of average worker</td>
<td>100% of average worker</td>
<td></td>
</tr>
<tr>
<td></td>
<td>no children</td>
<td>2 children</td>
<td>no children</td>
</tr>
<tr>
<td>A</td>
<td>63</td>
<td>64</td>
<td>75</td>
</tr>
<tr>
<td>B</td>
<td>64</td>
<td>64</td>
<td>79</td>
</tr>
<tr>
<td>C</td>
<td>75</td>
<td>64</td>
<td>42</td>
</tr>
<tr>
<td>D</td>
<td>87</td>
<td>44</td>
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<th></th>
<th>60 months</th>
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<tr>
<td></td>
<td>67% of average worker</td>
<td>100% of average worker</td>
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<tr>
<td></td>
<td>no children</td>
<td>2 children</td>
<td>no children</td>
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<tr>
<td>A</td>
<td>16</td>
<td>21</td>
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<tr>
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<tr>
<td>D</td>
<td>40</td>
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</table>

Notes:
All entries are in percent and are calculated as averages over the net replacement rates when the family qualifies for cash housing assistance or social assistance “top ups” and the case when it does not. For married couples the percentage of average worker relates to the previous earnings of the “unemployed” spouse only, the second spouse is assumed to be “inactive” with no earnings and no recent employment history in a one-earner couple and to have full-time earnings equal to 67\% of average worker in a two-earner couple.

<table>
<thead>
<tr>
<th>Type</th>
<th>Prior distribution</th>
<th>Posterior max.</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>std. error</td>
<td>mode</td>
</tr>
<tr>
<td>(\varsigma)</td>
<td>gamma</td>
<td>1.50</td>
<td>0.40</td>
</tr>
<tr>
<td>(\phi)</td>
<td>gamma</td>
<td>2.00</td>
<td>0.40</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>beta</td>
<td>0.60</td>
<td>0.05</td>
</tr>
<tr>
<td>(\eta)</td>
<td>beta</td>
<td>0.50</td>
<td>0.05</td>
</tr>
<tr>
<td>(\alpha_w)</td>
<td>beta</td>
<td>0.50</td>
<td>0.10</td>
</tr>
<tr>
<td>(\omega)</td>
<td>gamma</td>
<td>1.00</td>
<td>0.50</td>
</tr>
<tr>
<td>(\rho_\beta)</td>
<td>beta</td>
<td>0.85</td>
<td>0.10</td>
</tr>
<tr>
<td>(\rho_\zeta)</td>
<td>beta</td>
<td>0.85</td>
<td>0.10</td>
</tr>
<tr>
<td>(\rho_\rho)</td>
<td>beta</td>
<td>0.85</td>
<td>0.10</td>
</tr>
<tr>
<td>(\rho_\eta)</td>
<td>beta</td>
<td>0.85</td>
<td>0.10</td>
</tr>
<tr>
<td>(\rho_\sigma)</td>
<td>beta</td>
<td>0.85</td>
<td>0.10</td>
</tr>
<tr>
<td>(\rho_\omega)</td>
<td>beta</td>
<td>0.85</td>
<td>0.10</td>
</tr>
<tr>
<td>(\rho_\sigma)</td>
<td>beta</td>
<td>0.85</td>
<td>0.10</td>
</tr>
<tr>
<td>(\sigma_\beta)</td>
<td>inv. gamma</td>
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<td>inf</td>
</tr>
<tr>
<td>(\sigma_\zeta)</td>
<td>inv. gamma</td>
<td>0.01</td>
<td>inf</td>
</tr>
<tr>
<td>(\sigma_\rho)</td>
<td>inv. gamma</td>
<td>0.10</td>
<td>inf</td>
</tr>
<tr>
<td>(\sigma_\sigma)</td>
<td>inv. gamma</td>
<td>0.10</td>
<td>inf</td>
</tr>
<tr>
<td>(\sigma_\eta)</td>
<td>inv. gamma</td>
<td>0.20</td>
<td>inf</td>
</tr>
<tr>
<td>(\sigma_\sigma)</td>
<td>inv. gamma</td>
<td>0.01</td>
<td>inf</td>
</tr>
<tr>
<td>(\sigma_\sigma)</td>
<td>inv. gamma</td>
<td>0.01</td>
<td>inf</td>
</tr>
</tbody>
</table>
Table 3
Comparison of standard deviations, correlation and autocorrelation coefficients

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation</th>
<th>Standard deviation relative to standard deviation of $Y$</th>
<th>Correlation with output</th>
<th>First-order autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>0.0248 [0.0145]</td>
<td>1</td>
<td>1</td>
<td>0.9134 [0.7609]</td>
</tr>
<tr>
<td>$C$</td>
<td>0.0219 [0.0124]</td>
<td>0.8831 [0.8554]</td>
<td>0.8143 [0.7343]</td>
<td>0.8953 [0.7769]</td>
</tr>
<tr>
<td>$g$</td>
<td>0.0142 [0.0135]</td>
<td>0.5726 [0.9281]</td>
<td>-0.0089 [0.2742]</td>
<td>0.6424 [0.5859]</td>
</tr>
<tr>
<td>$U$</td>
<td>0.0096 [0.0128]</td>
<td>0.3871 [0.8806]</td>
<td>-0.4409 [-0.6941]</td>
<td>0.9478 [0.9609]</td>
</tr>
<tr>
<td>$V$</td>
<td>0.2200 [0.1559]</td>
<td>8.8710 [10.7411]</td>
<td>0.1078 [0.4336]</td>
<td>0.6948 [0.8896]</td>
</tr>
<tr>
<td>$w$</td>
<td>0.0335 [0.0363]</td>
<td>1.3508 [2.5001]</td>
<td>0.4612 [0.2781]</td>
<td>0.5296 [0.6848]</td>
</tr>
<tr>
<td>$Y^*$</td>
<td>0.0179 [0.0127]</td>
<td>0.8218 [0.8767]</td>
<td>0.3110 [0.5105]</td>
<td>0.8927 [0.7813]</td>
</tr>
</tbody>
</table>

Notes: the numbers without brackets refer to the moments generated by the model, whereas the numbers in brackets refer to those from the data.

Table 4
Conditional correlations for Beveridge curve

<table>
<thead>
<tr>
<th>Shock</th>
<th>Correlation between unemployment and vacancies</th>
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</thead>
<tbody>
<tr>
<td>Technology shock</td>
<td>-0.3509</td>
</tr>
<tr>
<td>Preference shock</td>
<td>-0.1900</td>
</tr>
<tr>
<td>Separation shock</td>
<td>0.9909</td>
</tr>
<tr>
<td>Vacancy posting cost shock</td>
<td>-0.5129</td>
</tr>
<tr>
<td>Bargaining power shock</td>
<td>0.0423</td>
</tr>
<tr>
<td>Government spending shock</td>
<td>-0.1533</td>
</tr>
<tr>
<td>Foreign output shock</td>
<td>-0.5555</td>
</tr>
<tr>
<td></td>
<td>Technology Preference Separation</td>
</tr>
<tr>
<td>-------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td><strong>Upon impact</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Y</strong></td>
<td>96.14</td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>17.25</td>
</tr>
<tr>
<td><strong>U</strong></td>
<td>0.00</td>
</tr>
<tr>
<td><strong>V</strong></td>
<td>2.45</td>
</tr>
<tr>
<td><strong>w</strong></td>
<td>4.00</td>
</tr>
<tr>
<td><strong>Y</strong></td>
<td>86.21</td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>37.88</td>
</tr>
<tr>
<td><strong>U</strong></td>
<td>0.83</td>
</tr>
<tr>
<td><strong>V</strong></td>
<td>2.07</td>
</tr>
<tr>
<td><strong>w</strong></td>
<td>10.77</td>
</tr>
<tr>
<td><strong>Y</strong></td>
<td>70.27</td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>38.37</td>
</tr>
<tr>
<td><strong>U</strong></td>
<td>1.20</td>
</tr>
<tr>
<td><strong>V</strong></td>
<td>1.89</td>
</tr>
<tr>
<td><strong>w</strong></td>
<td>13.78</td>
</tr>
<tr>
<td><strong>Y</strong></td>
<td>59.32</td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>28.51</td>
</tr>
<tr>
<td><strong>U</strong></td>
<td>1.11</td>
</tr>
<tr>
<td><strong>V</strong></td>
<td>1.88</td>
</tr>
<tr>
<td><strong>w</strong></td>
<td>13.82</td>
</tr>
</tbody>
</table>

Table 5
Variance decomposition (in %)
Figure 1
Prior and posterior densities
Notes: grey line – prior distributions, solid black line – posterior distribution, dashed black line – posterior modes.
Figure 2
Impulse responses to a one standard deviation shock
Notes: black line – posterior mean, dashed black lines – bounds of a 90% highest probability density interval. All impulse responses are reported as percentage deviations from the steady state.
Figure 3
Historical decomposition
Log-linearized model equations

This appendix lists all log-linearized model equations. Variables in letters with hat denote their log deviations from the deterministic steady state denoted by letters without time subscript.

Capital accumulation: \( \dot{K}_{t+1} = (1 - \delta)K_t + \delta\dot{I}_t \)
Consumption Euler equation: \( \dot{\lambda}_t + \hat{p}_t = \beta(r\hat{r}_{t+1} + (1 - \delta)p'_{t+1}) + \hat{\lambda}_t \) where \( \hat{\lambda}_t = \hat{\varepsilon}_{\beta,t} - \zeta \hat{C}_t \)
Optimal allocation of consumption expenditures: \( \hat{C}_t = \alpha' \hat{C}_{H,t} + (1 - \alpha')\hat{C}_{F,t} \)
Domestic consumption goods: \( \hat{C}_{H,t} = \hat{C}_t - \hat{p}_{H,t} \)
Exported consumption goods: \( \hat{C}_{F,t} = \hat{C}_t - \hat{p}_{F,t} \)
Optimal allocation of investment expenditures: \( \hat{I}_t = \alpha' \hat{I}_{H,t} + (1 - \alpha')\hat{I}_{F,t} \)
Domestic investment goods: \( \hat{I}_{H,t} = \hat{I}_t + \hat{p}_t - \hat{p}_{H,t} \)
Exported investment goods: \( \hat{I}_{F,t} = \hat{I}_t + \hat{p}_t - \hat{p}_{F,t} \)
Matching: \( \hat{M}_t = a\hat{U}_t + (1 - a)\hat{V}_t \)
Probability of finding a job: \( \hat{s}_t = \hat{M}_t - \hat{U}_t \)
Probability of finding a worker: \( \hat{q}_t = \hat{M}_t - \hat{V}_t \)
Labour market tightness: \( \hat{\theta}_t = \hat{V}_t - \hat{U}_t \)
Employment to unemployment link: \( \hat{N}_t = -\frac{U}{1 - \hat{U}_t} \)
Employment stock: \( \hat{N}_t = (1 - \rho)\hat{N}_{t-1} + \frac{M}{N} \hat{M}_{t-1} - \rho\hat{p}_t \)
Production function: \( \hat{Y}_t = \hat{Z}_t + a\hat{K}_t + (1 - \alpha)\hat{\lambda}_t \)
Firms’ discount factor: \( \beta_{t+1} = E_t(\hat{\lambda}_{t+1} - \hat{\lambda}_t) \)
Rental rate of capital: \( \hat{r}_t = \hat{m}\hat{c}_t + \hat{d}_t - \hat{K}_t \)
Markup equation: \( \hat{m}\hat{c}_t = \hat{p}_{H,t} \)
Asset value of a worker to the firm:
\[
J(\hat{\lambda}_t, \hat{\lambda}_{t+1}) = (1 - \alpha)\frac{1}{\mu N}(\hat{Y}_t + \hat{\lambda}_t) - w(\hat{\lambda}_t + \hat{\lambda}_{t+1})
+ (1 - \rho)\frac{\beta J(\hat{\lambda}_t + E_t(\hat{\lambda}_{t+1} + \hat{\lambda}_{t+1})) - \beta J\hat{p}_{t+1}}{\delta}
\]
Vacancy posting cost: \( \hat{\kappa}_t = \hat{q}_t - \hat{p}_{H,t} + E_t(\hat{\beta}_{t+1} + \hat{\beta}_{t+1}) \)
Real wage Nash bargaining:
\[
\hat{w}_t(N_t + \hat{\lambda}_t + \hat{\lambda}_{t+1}) = (1 - \eta)(b(\hat{\lambda}_t + \hat{\lambda}_{t+1}) + \kappa^L \frac{N_t + \phi N_{t+1}}{(1 + \phi)\lambda} (N_t + \phi N_{t+1}))
+ \eta \left( \frac{1}{\mu N}(\hat{Y}_t + \hat{\lambda}_t + \hat{m}\hat{c}_t + \hat{\eta}_t) + \beta s J(\hat{\lambda}_t + \hat{\lambda}_{t+1}) + \hat{p}_{H,t} + E_t(\hat{\beta}_{t+1} + \hat{\beta}_{t+1}) \right)
- \eta \left( b + \kappa^L \frac{N_t + \phi N_{t+1}}{(1 + \phi)\lambda} \hat{\eta}_t \right)
\]
Evolution of real aggregate wage: $\hat{w}_t = \alpha_w \hat{w}_t^N + (1 - \alpha_w)\hat{w}_{t-1}$

Government budget constraint: $g(\hat{g}_t + \hat{p}_{H,t}) + bU\hat{U}_t = T\hat{T}_t$

Foreign output: $Y_t^* = \rho_s Y_{t-1}^* + e_s$

Bilateral real exchange rate: $\hat{Q}_t = \hat{p}_{F,t}$

International risk sharing: $\hat{Q}_t = \xi(\hat{C}_t - \hat{C}_t^*)$

Exports demand: $\hat{X}_t = Y_t^* + \omega(\hat{p}_{F,t} - \hat{p}_{H,t})$

Resource constraint: $Y_t^* = C_{H,t} \hat{C}_{H,t} + I_{H,t} \hat{I}_{H,t} + g\hat{g}_t + \kappa^V(\hat{k}_{H,t}^v + \hat{V}_t) + X\hat{X}_t$

World market clearing constraint: $\hat{C}_t = Y_t^*$

Preference shock: $\hat{\epsilon}_{\beta,t} = \rho_{\beta} \hat{\epsilon}_{\beta,t-1} + e_{\beta,t}$

Productivity shock: $\hat{z}_t = \rho_{z} \hat{z}_{t-1} + e_{z,t}$

Separation shock: $\hat{p}_t = \rho_{p} \hat{p}_{t-1} + e_{p,t}$

Workers’ bargaining power: $\hat{\eta}_t = \rho_{\eta} \hat{\eta}_{t-1} + e_{\eta,t}$

Vacancy posting cost shock: $\hat{k}_{t}^v = \rho_{v} \hat{k}_{t-1}^v + e_{v,t}$

Government expenditure shock: $\hat{g}_t = \rho_{g} \hat{g}_{t-1} + e_{g,t}$