Persistent gaps between purchasing power parities and exchange rates under the law of one price: a puzzle (partly) explained?

Leon Podkaminer*

Submitted: 21 November 2012. Accepted: 4 April 2013

Abstract
Simple general equilibrium (exchange) models of intra-EU trade generate equalized relative prices of consumer tradables. Simultaneously, more dispersed domestic relative prices of consumer non-tradables are generated. Consequently, the purchasing power parity/exchange rate ratios may move further away from unity. The PPP/ER discrepancies may widen even when there are no impediments to free trade. While the models force the relative prices of tradables into uniformity, in reality these prices are persistently dispersed across countries. This does not vitiate the law of one price though. The retail prices of tradables depend also on prices of the local (non-tradable) inputs (e.g. distribution services). Prices of tradables net of costs of services (unobservable) may tend to obey the law of one price. The observed (retail) prices of tradables may not show this tendency even under free and competitive foreign trade because such a trade can widen the gaps between prices of non-tradables internationally.

Keywords: law of one price, purchasing power parities, exchange rates, general equilibrium modeling, cross-country systems of consumer demand functions

JEL: F11, F15, D12, D58, F31, O57

* The Vienna Institute for International Economic Studies; the School of Administration in Bielsko-Biała, Polish Academy of Sciences, the Institute of Economic Science; e-mail: podkaminer@wiiw.ac.at.
1. Introduction

Discrepancies between exchange rates (ER) and purchasing power parities (PPP) are primarily attributed to the presence of non-tradable goods. It is acknowledged that non-tradables do not have to obey the law of one price, at least directly. Because of this prices of non-tradables are assumed to be relatively free to vary in relation to prices of tradable goods across countries for extended periods of time. Thus prices of non-tradables entering the PPP calculations can drive a wedge between the PPP and ER. As is well known, demand for services rises with affluence and this fact is reflected in services (some of which tend to be non-tradable) being expensive relative to tradable goods in rich countries. Thus the PPP-ER gaps tend to be particularly large when it comes to comparing rich with relatively poor countries.

The law of one price which stipulates the tendency for equalization of prices of tradable goods internationally has long been (since at least David Ricardo) the work horse of the pure theory of international trade (see e.g. Froot, Kim, Rogoff 1995). But more recent studies seem to question the universal validity of the law. Contemporary literature abounds on factors ignored in pure trade theory such as transaction, distribution and transportation costs, cross-country differences in indirect taxation, impacts of competition imperfections (e.g. ‘pricing to market’, persistence of high mark-ups on foreign trade and domestic distribution activities), policy-related barriers to trade (tariffs, quota, regulations), etc. Obstfeld and Rogoff (2000) and Taylor and Taylor (2004) review a good deal of that literature. All these impediments to frictionless international trade are undoubtedly there and must be reflected in tradable goods’ prices failing to equalize internationally. As documented (e.g. in Anderson, van Wincoop 2004), the broadly defined trade-cost content of prices of goods traded internationally tends to be enormous. Moreover, evidence is growing that the law does not seem to operate even in the European Union – i.e. in a geographically compact area of sustained economic (and partly, monetary) integration; see e.g. Allington, Kattuman, Waldmann (2004); Dreger et al. (2007); Wolszczak-Derkacz, De Blander (2009). It turns out that prices of comparable tradable goods remain dispersed across the EU: formal statistical tests generally fail to detect their convergence. It is therefore quite reasonable to attribute, at least partly, the existence (and persistence) of gaps between PPP and ER also to the non-satisfaction of the law of one price with respect to the tradable goods. Of course, the price convergence across the OECD must be even weaker. Common currency has diminished the cross-country volatility of prices in the euro area (Foad 2008) while arbitrage pressures are stronger in the EU countries than across OECD (Méjean, Schwellnus 2009).

Many questions remain unanswered. In particular, why are prices of similar commodities dispersed even among countries that are highly integrated, with apparently free trade; unobstructed and otherwise competitive-looking. Is something fundamentally wrong with the law of one price – and with the neoclassical general equilibrium analysis in general? The question asked (and answered) in this note is much more modest – though eventually it might relate to the former

---

1 EU has been uniquely integrated as compared with e.g. OECD area or the world at large. It is quite clear that the law must be expected to operate much more vigorously in the EU than across OECD, or globally. Similarly, prices are expected to be less dispersed across the regions of small rather than large countries. Empirical evidence (e.g. PENN World Tables) abounds on the failure of the law to operate globally (see e.g. Heston 2004). Of course, there are numerous studies concerned with equalization of prices of selected standardized commodities (as well as financial products) traded internationally on specialized exchanges.
one. This question is: assuming the law of one price does equalize prices of tradable goods at no cost and instantaneously, would the gaps between purchasing power parities and exchange rates narrow? The answer to this question turns out to be ‘not necessarily’. Even under conservatively neoclassical assumptions the complete equalization of prices in international trade may actually result in a widening of PPP-ER deviations.

Section 2 briefly discusses the possibility of capturing the ER-PPP deviations in some simple models of the ‘pure’ theory of international trade. Section 3 outlines a simple general exchange equilibrium model of trade among the EU countries that is capable of reflecting the ER-PPP deviations. Section 4 discusses issues related to the data used for the estimation of the system of cross-country system of demand equations underlying the model. Section 5 presents the parameter estimates for a cross-country Almost Ideal Demand System (AIDS). Section 6 shortly discusses the solutions to the pan-European general exchange equilibrium model. Section 7 shows that the PPP/ER ratios following equalization of relative prices of tradable goods quite often move away from unity. Section 8 concludes, briefly returning to the issue of non-equalization of prices of tradable goods actually observed.

2. The PPP-ER gaps in simple neoclassical models of international trade

In the simplest model of international trade there are two countries, each capable of producing the same two final homogeneous consumption goods (e.g. Ricardo’s ‘wine and cloth’). These goods are subject to free and costless exchange. Under the usual (neoclassical) assumptions (on endowments, technologies, preferences and competition) versions of this canonical model generate a unique relative price for the two goods. This price obtains internationally, as well as domestically in either country. Of course, besides equilibrating demand and supply, this relative price leaves both countries with balanced trade. Being neoclassical, the model cannot say anything about absolute price levels in either country – and of course about the absolute magnitudes of the exchange rate and the purchasing power parity. It is quite obvious though that if the absolute prices in one country’s currency happen to be multiples of absolute prices in the other country’s currency (\(m\) being the multiple) then the exchange rate would simply be \(1/m\) – as would also be the purchasing power parity, irrespective of the differences in the countries’ consumption structures.\(^2\)

Thus, in the simplest neoclassical trade model there is no place for the emergence of a gap between PPP and ER. This is unsurprising since the model assumes the equalization of relative prices and does not allow for the presence of non-tradable goods.

Another simple model with some neoclassical features underlies a huge literature on the Balassa-Samuelson effect (BSE). That model also has two countries (a ‘home country’ and ‘the rest of the world’) producing two homogeneous goods – of which only one is considered tradable (see e.g. De Gregorio, Giovannini, Wolf 1994). Although the model underlying the BSE has been extensively referred to in exchange rate economics (see e.g. Sarno 2005), it is fraught with fundamental

\(^2\) Formally, PPP could be defined here as the geometric mean of the bilateral Paasche and Laspeyres price indices. When the relative prices of the two goods are the same in both countries, the Paasche and Laspeyres indices are both equal to \(1/m\) irrespective of the national consumption structures – with the PPP also equaling \(1/m\).
conceptual difficulties (see Podkaminer 2003; 2011). Of course this is not the place to reiterate the reasons why the model behind the BSE may not advance our understanding of the ER-PPP issue. It is sufficient to notice, that this model actually rules out international trade – though it is also assumed that the law of one price equalizes the price of the single tradable good. Specifically, when one good – by assumption precisely the same tradable good – is produced in each country, there is no reason to engage in trade: Portugal does not have any reason to trade its ‘wine’ for the same wine supplied by England. Moreover, unless there is some third commodity (e.g. gold), each country would end up with domestic consumption equal to domestic supply. How then would the price equalization come about? And, relative to the price of which commodity would the prices of tradable good equalize? Certainly the tradable goods’ equalization cannot be understood as being relative to the prices of the non-tradable good in individual countries. That the price of tradable goods relative to the price of non-tradable services varies substantially and systematically over time and space (and in relation to the income level) is indisputable.

Concluding, for a (neoclassical) model of international trade to make sense it has to have at least two countries and two tradable goods. Additionally, for that model to allow for the emergence of discrepancies between PPP and ER it has to have at least one non-tradable good.

3. An outline of the model

The model considered below is estimated for 25 countries all of which are EU member states (excluding, for reasons given later on, two mini-states: Luxembourg and Malta). The two tradable goods are defined as ‘non-durable consumer goods’ and ‘durable consumer goods’, and the item ‘non-tradable services’ is identified as ‘household consumption other than consumption of non-durable and durable goods’ (to be defined in some detail shortly). Each country’s ‘average consumer’ in any specific year is characterized by six values: real per capita quantities of consumption of the three items defined above ($Q_n$, $Q_d$, $Q_s$) and their respective partial purchasing power parities ($p_n$, $p_d$, $p_s$). The values come (after some modifications) from Eurostat which oversees the European comparison project. The PPPs are calculated relative to the EU-27 average levels, and the real quantities consumed, measured at PPPs, are computed accordingly.

The relative price of tradable goods (durable vs. nondurable, defined as $\pi = p_d / p_n$) is far from being the same across countries. The coefficient of variation of national relative prices $\pi$ varies from 0.173 in 1999 (the earliest year for which the consistent unified ECP data are now reported) to 0.149 in 2004 and 0.094 in 2011. Clearly, there has been some convergence, far from complete so far, in relative prices $\pi = p_d / p_n$ across the EU.

It may be noticed that non-uniqueness of the relative price $\pi$ could also be due to the fact that the compositions of both aggregates may significantly vary from country to country. That is, for

3 Rigorous derivation of the BSE requires a number of daring assumptions – often balanced on a knife’s edge (with infinitesimal changes in the assumed parameters resulting in a collapse of the model’s desired properties). One of these assumptions presumes neutral technical change, another rules out application of intermediate inputs in production. Apart from being theoretically questionable in their own right, these assumptions are unlikely to be satisfied in reality. Moreover, econometric studies offer little in way of support for BSE. This empirical research is summarized in Égert (2007), who states that ‘...our estimation results provide the obituary notice for the Balassa-Samuelson effect’ (Égert 2007, p. 1).
instance, durables in some countries may consist primarily of specific goods that are relatively rare in some other countries. For the purposes of this note, we shall have to disregard the existence of such compositional effects and treat the dispersion in $\pi$ as a sign of non-satisfaction of the law of one price, or the failure – for whatever reason – of inter-EU trade to equalize the relative price of the two homogeneous tradable goods.\textsuperscript{4} In terms of the pure theory of international trade the observed bundles of tradable goods (consisting of durables and nondurables) are distributed across countries Pareto-inefficiently. The assumption is that that distribution could have been improved through additional exchange among countries, resulting in equalization of relative prices of both tradable goods across countries.

Assume, conventionally, that the country $k$ (identified with its average or ‘representative’ consumer) is characterized by a system of well-behaved demand functions:

$$
Q_n^k = Q_n^k(M^k, p_n^k, p_d^k, p_s^k)
$$

$$
Q_d^k = Q_d^k(M^k, p_n^k, p_d^k, p_s^k)
$$

$$
Q_s^k = Q_s^k(M^k, p_n^k, p_d^k, p_s^k)
$$

where $M^k = p_n^k Q_n^k + p_d^k Q_d^k + p_s^k Q_s^k$ is the total per capita nominal income (or total per capita household consumption expenditure) in country $k$.

Being well-behaved (and thus in particular homogeneous of degree zero in prices and income), the above demand equations can be written equivalently in terms of two relative prices $\pi^k = p_d^k/p_n^k$ and $P^k = p_s^k/p_n^k$.

$$
Q_n^k = Q_n^k(M^k*, \pi^k, P^k)
$$

$$
Q_d^k = Q_d^k(M^k*, \pi^k, P^k)
$$

$$
Q_s^k = Q_s^k(M^k*, \pi^k, P^k)
$$

where $M^k* = Q_n^k + \pi^k Q_d^k + P^k Q_s^k$

Observe that $P^k = p_s^k/p_n^k$ denotes country’s $k$ relative price of non-tradable services in terms of non-durables price.

Let us now assume that countries engage in costless (and otherwise unrestricted) trade, exchanging some domestically-produced non-durables in return for some durables available to some other countries. Under the standard assumptions this trade, beneficial to all parties involved, would be concluded (with the help of the Walrasian auctioneer) at a single relative price $\pi = p_d/p_n$

\textsuperscript{4} The cross-country differences in relative ratios derived from the purchasing power parity aggregates (such as $\pi = p_d/p_n$ tend to be smaller than the PPP ratios for individual items (e.g. Crucini, Telmer, Zachariadis 2005). The non-satisfaction of the law of one price is actually more pronounced than the aggregates may suggest. For example, two countries may happen to have the same $\pi$, without having the same relative prices for disaggregate items.
leaving the total supply aggregated over all participating countries unchanged and the values of each country’s imports equal to the values of its exports.

Formally, the equalized relative price of the two tradable goods has to satisfy either of the following two equations:

\[ \sum_k L^k Q^k_n = \sum_k L^k Q^k_n (M^k, \pi, P^k) \]  
\[ \sum_k L^k Q^k_d = \sum_k L^k Q^k_d (M^k, \pi, P^k) \]  

where \( L^k \) stands for the population of country \( k \), and \( Q^k_j \) is the per capita availability (supply = consumption recorded) of good \( j \) \((j = n, d)\) recorded in country \( k \).

The equations (1) contain 25 unknowns \( P^k (k = 1, ..., 25) \) which are the national relative prices of services in terms of the national relative price of non-durable tradable good plus one single unknown \( \pi \). It is postulated here that in addition to (1) the 26 unknowns satisfy 25 equations

\[ Q^k_{s} = Q^k_{s} (M^k, \pi, P^k) \quad k = 1, ..., 25 \]  

The satisfaction of (2) keeps each country’s demand for non-tradable services at the level actually reported (\( Q^k_{s} \)).

Suppose the system of equations (1)–(2) is numerically specified and solved, yielding the equalized relative price of the two tradable goods \( \pi^e \), the modified (through exports and imports) quantities of tradable goods consumed and the vector of national relative equilibrium prices of services, \( P^k^e \). Given the solution to (1)–(2), it is straightforward to calculate the purchasing power parities for individual countries for the post-price-equalizing-trade situation. However, direct comparison of these post-trade PPPs with the original ones (Eurostat’s) is not possible. The original PPPs are defined in relation to absolute prices (normalized at 1 for the average for the entire EU) while the PPPs derived from the solutions to (1)–(2) are defined in terms of relative prices (though these are also normalized at 1 for the entire EU). However, comparability of pre-trade and post-trade PPPs is still possible. The original Eurostat data can always be presented in terms of the relative \((\pi^k, P^k)\) rather than absolute \((p^k_d, p^k_n)\) prices. Of course, comparisons of the pre-and post-trade PPPs do not appear to convey some obviously informative information. However, things get more interesting if one observes that the solutions to (1)–(2) – as well as the original data presented in relative terms – can be used to calculate the PPPs for the aggregate consisting of the tradable goods only. The following ratio

\[ \frac{PPP_{all \ consumption}}{PPP_{tradable \ goods}} \] 

is then interpreted as the PPP/ER ratio obtaining in the neoclassical world described by the model defined above.

---

5 The satisfaction of one of (1) implies the satisfaction of the other.
The ratio (3) can be computed also for the original situations reflected by the Eurostat data (i.e. prior to price-equalizing free trade). It must be reiterated that being neoclassical, the model in question is incapable of saying anything about price levels obtaining post-trade in individual countries, purchasing power parities or values of the exchange rates. Needless to say, it also cannot possibly allow for the effects of capital (or money) flows which certainly affect exchange rates ‘in real life’—outside the neoclassical theory. Nor can it allow for imbalanced trade among countries (as such trade would imply the necessity to allow for capital flows of some sort). Moreover, in keeping with the tradition of the pure theory of international trade, it abstracts from the existence of goods other than consumption ones. Neither capital goods nor intermediate inputs are considered. It also ignores any trade with the rest of the world: our 25 EU countries represent the whole world here.

Of course, in reality the EU countries have extensive (and diverse) trade links with the outside world. Their domestic prices are differently affected by their individualized trades with the rest of the world. (For example, Campa and González-Mínguez (2006) show that various degrees of openness to the rest of the world imports is the most important factor explaining the differences in pass-through of exchange rates into import prices of euro area countries). But the stylized model (1)–(2) serves the purpose of eliciting the effects of free (‘neoclassical’) trade only. As such it cannot allow for any country-specific differentiation of trades (within EU, or outside it).6

To repeat, the model (1)–(2) rules out any ‘imperfections’ (trade costs, barriers to trade, differences in indirect tax rates, oligopolistic practices etc.) that could possibly interfere with the operation of the law of one price.

The strategy followed in this paper should by now be somewhat clearer. The ultimate goal is to compute the ratios (3) for the pre- and post-price-equalizing-trade situations for a sufficiently long span of time and then to check whether price-equalizing free trade would be always moving the ratios (3) towards unity. Podkaminer (1999), working with the then available European comparison project data for 1990, found many instances of the ratio (3) actually moving away from unity – i.e. the evidence that free trade and price equalization might actually enlarge the deviations between exchange rates and purchasing power parities. But that outcome may have been due to imperfections with the data in many European countries then undergoing major changes along with very high rates of inflation and continuing presence of prices being officially administered, with shortages/rationing of consumer goods and services all distorting the reported data (Podkaminer 1982).

4. The data issues

To specify (1)–(2) one would need to have some knowledge of systems of consumer demand functions for 25 EU member states. This sounds quite straightforward, but in fact is not. Utilizing national statistics that could produce 25 separate national systems of the consumer demand functions of the sort needed would involve a gigantic amount of work. Moreover, the national demand functions would have to be defined for comparable goods. To get around this problem some applied work whose aim is to estimate, with reasonable precision, a cross-country system

---

6 In principle it would be possible to include all countries of the world in (1)–(2). But a specification of such a global equilibrium model would require sufficiently detailed (and reasonably reliable) data for all countries. Besides, the free-and-costless trade assumption which may approximate the EU reality, does not seem realistic for the world at large.
of demand functions is advisable. This is the approach developed by the late Professor Henri Theil (see Theil, Suhm 1981; Theil, Clements 1987; Fiebig, Seale, Theil 1988; Clements, Selvanathan 1994) – and followed here. The approach assumes the existence of a single universal system of demand functions characterizing different national ‘representative consumers’. Conventionally, the national price and quantity data for a given period are assumed to be generated by a system of demand functions, the same for each of the countries considered. (This rules out ‘identification problems’). It is then proposed to estimate the cross-country systems of demand functions, using cross-section data on national quantities and prices taken from the international comparison projects. The data available from such projects typically display large cross-country variations in both income levels and relative prices. Besides, such data implies comparability of national volume and price indices. Of course, the diversity in the cross-section data available from the comparison projects (and the internal consistency of that data and its cross-country comparability) come at a cost. The projects’ methods of collecting and processing (aggregation) of information do not cease to be debated (and occasionally revised). Nonetheless, the reliance on data eventually generated by the comparison projects for the specific goal of estimating the cross-country systems of the demand function does not seem to be any less acceptable than relying on these data for any other purpose. Of course it must be reiterated that this tacitly assumes a belief into the objective existence of some patterns of consumer demand formation that are worthy of being qualitatively examined and a belief that the patterns in question are fairly universal over time and space. The latter belief has a long tradition in the applied consumer economics (Houthakker 1957). Of course, (1)−(2) could be specified with functionally different systems of demand functions for individual countries (should this note works with a subset of the country data available from Eurostat’s European comparison project (ECP henceforth).8

Due to the anomalous price and income conditions observed in Luxembourg, this country is not considered in the analysis below. Luxembourg’s very high income level happens to be combined with a relative price (goods/services) that is not much different from that recorded in that country’s much less affluent neighbours. Luxembourg does not therefore conform to the regularity. This anomaly can be explained by the country’s tiny size and location between neighbours characterized by much lower income levels. Prices of goods and services, including housing rents, recorded in Luxembourg cannot diverge radically from those prevailing in towns or shopping centers located a few miles away – just across the (nearby) borders. The opposite irregularity (comparatively high relative prices of services at a relatively low income level) can be detected in some small countries highly dependent on income from foreign tourism (e.g. Malta). As such, this country is also excluded from the analysis below.

At present the ECP reports purchasing power parities and nominal, as well as ‘real’ (PPP-adjusted), quantities of apparently tradable consumer goods (sub-divided into durable consumer goods, semi-durable consumer goods and non-durable consumer goods) and consumer services (assumed to be non-tradable). The first, minor, problem is that ECP offers data for three sub-categories of consumer goods – while one could be perfectly satisfied with data for two sub-categories. However, the aggregation of data for durables and semi-durables does not entail any

7 Of course, if the preferences of representative consumers of different countries were much different, (1)−(2) could be specified with functionally different national systems of demand functions.
9 Housing’ (with water, gas and electricity supply), ‘health’ and ‘education’ – the consumer service items that are hardly tradable internationally – accounted for 72% of total consumer expenditure in the EU in 2002 (Eurostat/OECD 2004).
serious difficulty. A more serious problem is that for most countries the aggregates for the two consumer items (goods and services) differ appreciably from reported data on ‘national household final consumption expenditure’. Just to illustrate this point, consider the ‘raw’ ECP data for Germany and Spain in 2005. The nominal German per capita expenditures on the consumption of consumer goods and services equaled 7809 and 7440 euro respectively (15 249 euro in total) – while the per capita nominal household final consumption was as much as 15 593 euro. The opposite situation was reported for Spain, where the nominal per capita expenditures on consumer goods and services were 5427 and 7132 euro respectively (12 559 euro in total) – while the nominal per capita household final consumption reportedly equaled only 11 935 euro. These discrepancies are attributable primarily to the households’ consumption realized abroad (in the German case), or to foreigners’ consumption (in the Spanish case). Earlier Eurostat reports contained all information (nominal and real quantities as well as purchasing power parities) pertaining to the item called ‘net purchases abroad’. That item tightly corresponded to the discrepancies between the household final consumption expenditure and the sum of the consumer goods and services (see e.g. Eurostat/OECD 2004). Whatever the nature of the discrepancies in question, it is quite obvious that they have to be consistently removed from the ECP data before the estimation of the conventional systems of households’ demand functions. To get rid of these discrepancies it is assumed that in each country instance the discrepancies (in volumes, values and the purchasing power parities) reflect net purchases abroad and that the transactions involving foreigners are only in consumer services. In other words it is assumed that while abroad, the tourists buy (from locals) only services. The households in the tourism-importing countries (such as Germany) are thus assumed to consume more of services than reported by the ECP for Germany – and as much of the consumption goods as reported. Conversely, households in the tourism-exporting countries are assumed to consume less of services than reported by ECP for their countries – and unchanged quantities of goods. Further, it is assumed that while the purchasing power parities of the consumer goods are equal to the purchasing power parities of the households’ final consumption expenditure on goods, the purchasing power parities of households’ final consumption of services remain to be assessed – taking into account services consumed both domestically and abroad. Finally, it is postulated that the real quantities of services actually entering households’ final consumption expenditure (and the corresponding services’ purchasing power parities) must be consistent with the overall purchasing power parities for the household final consumption expenditure. (The latter are of course reported by ECP).

Under these assumptions it is relatively easy to compute the adjusted purchasing power parities (PPPₕ) for the consumer services included in the aggregate household final consumption expenditure.

It turns out that the adjusted PPPₕ do not diverge meaningfully from the original ECP (unadjusted) ones. For example, the adjusted German PPPₕ for 2005 is 1.0241 rather than the original 1.0333 and Spain’s PPPₕ for that year are 0.9319 and 0.9262 respectively. But the real adjusted real quantity Qₕ for Germany in 2005 is about 7600 euro rather than the original 7200. The respective Qₕ quantities for Spain are 6930 and 7700 euro. Undoubtedly, the adjusted quantities seem more likely than the original ones: there is little doubt that being much richer, the average German must in fact consume more services than the average Spaniard (see also Podkaminer (2011) on the tendency of services’ share to rise with income).
5. An Almost Ideal Demand System (AIDS) fits the data well

After some experimentation with popular functional forms for the system of demand functions, a simplified version of the ‘classical’ Almost Ideal Demand System (Deaton, Muellbauer 1981) was eventually selected.

Arithmetically, the simplified AIDS for the three-commodity economy is compactly represented by three demand equations:

\[ Q_n = \left( \frac{M}{p_n} \right) [\alpha_n + \beta_n (\log \left( \frac{M}{M^0} \right) - \Pi)] \]

\[ Q_d = \left( \frac{M}{p_d} \right) [\alpha_d + \beta_d (\log \left( \frac{M}{M^0} \right) - \Pi)] \] (4)

and

\[ Q_s = \left( \frac{M}{p_s} \right) [\alpha_s + \beta_s (\log \left( \frac{M}{M^0} \right) - \Pi)] \]

\( Q_n, Q_d \) and \( Q_s \) are real quantities of household consumption of non-durables, durables (with semi-durables) and services respectively, \( M \) is nominal per capita household expenditure, \( M^o \) is the scaling constant identified – in our case – with average per capita real (which by construction is equal to the nominal) household expenditure for the entire EU-27, \( p_n, p_d \) and \( p_s \) are the purchasing power parities (by definition playing the role of prices) for the two consumer goods and services respectively, log refers to the natural logarithm, and \( \Pi \) is the overall (Richard Stone’s) price index defined as

\[ \Pi = \alpha_n \log(p_n) + \alpha_d \log(p_d) + \alpha_s \log(p_s) \] (5)

Finally, \( \alpha_n, \alpha_d, \alpha_s, \beta_n, \beta_d, \beta_s \) are the parameters to estimate from (4) (with (5) being plugged into (4)).

Non-linear full information maximum likelihood (FIML\textsuperscript{10}) was applied, in two stages, for the estimation of the parameters for consecutive years.\textsuperscript{11} In the first stage no restrictions on the parameters were imposed. It was then checked whether these first-stage estimates satisfied the adding-up and symmetry conditions, as required by the conventional choice theory. In the case considered the conditions can be tested by means of the Wald test \( H_0 \):

\[ H_0: (\alpha_n + \alpha_d + \alpha_s = 1; \beta_n + \beta_d + \beta_s = 0) \]

The test is passed with ‘flying colors’: the p-value for the F-statistic is higher than 0.750 for each year meaning that there are no reasons for rejecting \( H_0 \).

The second-stage estimates were obtained with the restrictions imposed on the parameters following the elimination of parameters \( \alpha_d \) and \( \beta_d \) (i.e. with \( \alpha_d = 1 - \alpha_n - \alpha_s \); \( \beta_d = -\beta_n - \beta_s \)).

\textsuperscript{10} Often used, FIML has well known (and desirable) properties, also in the case where prices are endogenous in (4).

\textsuperscript{11} As will be explained shortly it is not legitimate to pool together the data for separate years with the aim of running panel regressions.
 Persistent gaps between purchasing power parities...  

Table 1 reports the parameter estimates obtained in the second stage through the application of the non-linear FIML method to the data for consecutive years. Of course only the parameters for services and non-durables were estimated (and the values for the parameters for durables followed from the restrictions imposed).

The statistical quality of the parameter estimates (for services and non-durables) turns out to be very high. The p-value is smaller than $10^{-4}$ for each parameter estimate. Also, in most cases the ‘fit’, measured by adjusted $R^2$, is rather high (see Table 2)\textsuperscript{12}.

Two remarks are in order. Firstly, the negative values of $\beta_n$ (on average -0.2219) indicate that non-durables are ‘necessities’. Similarly as in the case of ‘food’ (which is included in non-durables), the share of non-durables in total consumer expenditure declines with income level, implying that Engel’s law applies to the whole non-durables aggregate also. But the average $\beta$ for services is positive and large (0.1662) indicating that services are ‘luxuries’. These conclusions, perhaps not quite novel, were also confirmed in the context of research on cross-country systems of demand functions (Podkaminer 1999; 2004; 2011).

Secondly, although the parameter estimates for consecutive years are on the whole quite similar, they are not identical. Moreover, they seem to follow some sustained tendencies, especially visible in the case of $\beta_n$ (generally declining over time) and $\beta_s$ (generally increasing over time). However, it should be noticed that the parameter estimates for different years cannot be expected to be the same. This follows from the fact that the results of ECP (or of any other international comparison project) for various years are not mutually comparable.

\textsuperscript{12} The quality of fit for the non-durables equation gets slightly worse for 2007−2009 and again for 2011. Data for 2011 must be treated as still provisional. One is free to speculate about consumer behaviour in 2007—2009 being distorted, as compared with ‘normal’ patterns, by abnormal macroeconomic developments (runaway consumer credit booms in many EU countries culminating in 2007, followed by credit crunches in 2008).
Purchasing power parities and real quantities for a given year and given country cannot be legitimately compared with the respective items even for the same country in a different year. The same incomparability principle applies to the measures of total real consumption (approximated by \( \log(M/M^\circ) - \Pi \)). In particular, average EU per capita household consumption (\( M^\circ \)) itself is a nominal magnitude. Due to this, the series of \( M^\circ \) for consecutive years reflects ongoing inflation. For example, \( M^\circ \) for 2003 is 11,800 euro (at purchasing power parities of 2003) while \( M^\circ \) for 2004 is 12,300 euro (at purchasing power parities of 2004). The implied growth rate of average EU per capita household consumption is 4.24%. But this rate reflects both inflation and the structural change (in both prices and real quantities consumed). The price index \( \Pi \), calculated separately for each year, allows one to make cross-country comparisons for the given year only. There is nothing in the definition, or construction, of \( \Pi \) which would suggest it could be used to deflate nominal consumption values for different years – even for the same country. It may be added that even though one does not quite know how to relate the price indices \( \Pi \) for consecutive years even at the overall EU level,\(^{13}\) one may safely assume that in most member states (and at the EU level) average per capita real consumption must have continued to grow (at least until 2007), even if one does not know precisely how to measure that growth in PPP terms. Given this observation, one should expect \( \beta \) to get larger in absolute terms over time. In other words, even if \( (\log(M/M^\circ) - \Pi) \) for a country happens to be the same in two years, its ‘true real’ value is likely to have been larger in the later year.

Finally, it is worth adding that large jumps in the parameter estimates occurred in 2005. This may be due to a change in the Eurostat methodology for the calculation of prices and volumes for education services (which constitute an important part of the service aggregate) which happened in 2005.

---

\(^{13}\) Because \( p_{se}, p_d \) and \( p_c \) for the whole EU are all unity by construction, \( (\log(M/M^\circ) - \Pi) \) for the average EU inhabitant is 0 in any year. Correspondingly, \( Q_s \) for that inhabitant is estimated as \( \alpha_s M^\circ \).
In any case it has to be reiterated that being incomparable over time, the ECP data for separate years must not be pooled together. Pooling the data – and then running panel regressions – is not legitimate. By the same principle there is little justification for formal modeling of the dynamics of the parameter estimates from Table 1.

6. The equilibria

For each year the system of equations (1)–(2) is specified and solved. The AIDS demand functions specified with the parameters from Table 1 are plugged into the right-hand sides of these equations. (To safeguard comparability with the original data, the right-hand sides of these equations include, additionally, the values of the residuals for the respective regressions).

The systems (1)–(2) are highly non-linear in the unknowns, but obtaining solutions to them (through repeated iterations) proceeds very quickly.

Table 3 illustrates the character of solutions to the model (1)–(2) for the odd-dated years only (this is merely to save on space). The first row reports un-weighted averages of the relative price of tradable goods \( \pi = p_d/p_n \) characterizing the original ECP data, the second row the un-weighted coefficients of variation of \( \pi \). The third row reports the equilibrium value of \( \pi \), i.e. the effect of the operation of the law of one price. The fourth row reports un-weighted averages of pre-trade relative prices of services (in terms of prices of non-durables) with the fifth reporting the respective coefficients of variation. The sixth and seventh rows refer to the post-trade relative prices of services.

Given the difficulties inherent in comparing the ECP data (and the data derived from them) over time, it is perhaps advisable to comment on the contents of Table 3 with some caution. However, it seems correct to highlight one fact, namely that in any year the equalization of the prices of tradable goods happens to be associated with the relative prices of services becoming more dispersed as compared with the original pre-trade situations. (The coefficients of variations from the seventh row are all larger than those from the fifth row).

The price solutions to (1)–(2) imply definite reallocations of consumption of the two tradable goods, with some countries exporting ‘surpluses’ of durables and some exporting ‘surpluses’ of

<table>
<thead>
<tr>
<th></th>
<th>1999</th>
<th>2001</th>
<th>2003</th>
<th>2005</th>
<th>2007</th>
<th>2009</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average pre-trade ( \pi )</td>
<td>1.1132</td>
<td>1.0790</td>
<td>1.0945</td>
<td>1.0960</td>
<td>1.0783</td>
<td>1.0602</td>
<td>1.0216</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.1732</td>
<td>0.1380</td>
<td>0.1602</td>
<td>0.1491</td>
<td>0.1324</td>
<td>0.1190</td>
<td>0.0943</td>
</tr>
<tr>
<td>Post-trade (equilibrium) ( \pi )</td>
<td>0.9687</td>
<td>0.9687</td>
<td>0.9732</td>
<td>0.9722</td>
<td>0.9761</td>
<td>0.9818</td>
<td>0.9780</td>
</tr>
<tr>
<td>Average pre-trade ( P )</td>
<td>0.7799</td>
<td>0.7984</td>
<td>0.8505</td>
<td>0.8619</td>
<td>0.8733</td>
<td>0.8819</td>
<td>0.8755</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.3282</td>
<td>0.3164</td>
<td>0.2289</td>
<td>0.2261</td>
<td>0.2130</td>
<td>0.2160</td>
<td>0.2242</td>
</tr>
<tr>
<td>Average post-trade ( P )</td>
<td>0.7728</td>
<td>0.7900</td>
<td>0.8419</td>
<td>0.8507</td>
<td>0.8619</td>
<td>0.8754</td>
<td>0.8761</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.3385</td>
<td>0.3310</td>
<td>0.2545</td>
<td>0.2463</td>
<td>0.2337</td>
<td>0.2307</td>
<td>0.2366</td>
</tr>
</tbody>
</table>
non-durables. As illustrated by Table 4, in the ‘old’ EU member states the sizes of these trades are rather small in relation to the domestic consumption levels originally reported. However, these magnitudes have been rather large in the ‘new’ member states (as well as in Spain, Greece, Ireland and Portugal). The interpretation of this fact seems fairly straightforward with ‘old’ (and also rich) EU member states having been much more integrated through mutual trade than the ‘new’ (or ‘cohesion’) member states. The ‘old’ countries have had time to integrate and would need only marginal additional adjustments (captured by the model) to arrive at the Walrasian equilibrium. This is not the case with the ‘new’ member states, whose consumption patterns still seem to require quite massive adjustments.

Interestingly, the models’ solutions suggest that less affluent countries would have exported significant quantities of non-durables which they actually consumed, in exchange for significant

Table 4
Exports and imports (negative) as percentage of domestic pre-trade consumption, associated with the solution to (1)–(2), selected years

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>0.8</td>
<td>-0.4</td>
<td>0.5</td>
<td>-0.2</td>
<td>-0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Bulgaria</td>
<td>-23.7</td>
<td>4.1</td>
<td>-15.4</td>
<td>5.1</td>
<td>-11.4</td>
<td>4.4</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>-29.3</td>
<td>7.7</td>
<td>-17.9</td>
<td>5.6</td>
<td>-4.7</td>
<td>1.4</td>
</tr>
<tr>
<td>Denmark</td>
<td>5.4</td>
<td>-3.5</td>
<td>3.5</td>
<td>-3.8</td>
<td>4.0</td>
<td>-4.2</td>
</tr>
<tr>
<td>Germany</td>
<td>3.6</td>
<td>-2.4</td>
<td>3.1</td>
<td>-2.2</td>
<td>2.2</td>
<td>-1.2</td>
</tr>
<tr>
<td>Estonia</td>
<td>-16.6</td>
<td>10.4</td>
<td>-11.2</td>
<td>9.8</td>
<td>-7.0</td>
<td>6.6</td>
</tr>
<tr>
<td>Ireland</td>
<td>8.7</td>
<td>-6.0</td>
<td>8.0</td>
<td>-6.7</td>
<td>8.8</td>
<td>-10.8</td>
</tr>
<tr>
<td>Greece</td>
<td>-9.6</td>
<td>4.8</td>
<td>-9.8</td>
<td>3.3</td>
<td>-3.1</td>
<td>1.1</td>
</tr>
<tr>
<td>Spain</td>
<td>-14.8</td>
<td>7.3</td>
<td>-8.6</td>
<td>4.9</td>
<td>-5.3</td>
<td>3.1</td>
</tr>
<tr>
<td>France</td>
<td>3.7</td>
<td>-2.0</td>
<td>1.2</td>
<td>-0.6</td>
<td>-0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>Italy</td>
<td>0.2</td>
<td>-0.2</td>
<td>1.4</td>
<td>-1.1</td>
<td>0.4</td>
<td>-0.3</td>
</tr>
<tr>
<td>Cyprus</td>
<td>-7.8</td>
<td>4.1</td>
<td>-1.4</td>
<td>0.8</td>
<td>1.1</td>
<td>-0.8</td>
</tr>
<tr>
<td>Latvia</td>
<td>-34.1</td>
<td>8.3</td>
<td>-22.8</td>
<td>9.0</td>
<td>-7.9</td>
<td>3.1</td>
</tr>
<tr>
<td>Lithuania</td>
<td>-34.7</td>
<td>5.5</td>
<td>-19.0</td>
<td>5.9</td>
<td>-10.4</td>
<td>2.4</td>
</tr>
<tr>
<td>Hungary</td>
<td>-16.1</td>
<td>7.5</td>
<td>-14.8</td>
<td>5.0</td>
<td>-3.5</td>
<td>1.2</td>
</tr>
<tr>
<td>Holland</td>
<td>3.6</td>
<td>-2.7</td>
<td>-4.0</td>
<td>2.8</td>
<td>-2.2</td>
<td>1.7</td>
</tr>
<tr>
<td>Austria</td>
<td>-0.8</td>
<td>0.6</td>
<td>-0.6</td>
<td>0.4</td>
<td>-0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>Poland</td>
<td>-38.1</td>
<td>3.7</td>
<td>-33.2</td>
<td>4.8</td>
<td>-25.2</td>
<td>2.0</td>
</tr>
<tr>
<td>Portugal</td>
<td>-4.2</td>
<td>4.1</td>
<td>-9.1</td>
<td>5.4</td>
<td>-6.4</td>
<td>4.4</td>
</tr>
<tr>
<td>Romania</td>
<td>-21.2</td>
<td>7.0</td>
<td>-19.3</td>
<td>4.3</td>
<td>-27.1</td>
<td>3.2</td>
</tr>
<tr>
<td>Slovenia</td>
<td>-6.2</td>
<td>3.2</td>
<td>-6.5</td>
<td>4.1</td>
<td>-3.4</td>
<td>1.4</td>
</tr>
<tr>
<td>Slovakia</td>
<td>-35.9</td>
<td>5.2</td>
<td>-31.5</td>
<td>3.1</td>
<td>-11.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Finland</td>
<td>2.6</td>
<td>-1.7</td>
<td>-1.3</td>
<td>0.9</td>
<td>1.6</td>
<td>-0.9</td>
</tr>
<tr>
<td>Sweden</td>
<td>5.2</td>
<td>-2.7</td>
<td>3.3</td>
<td>-2.1</td>
<td>2.3</td>
<td>-1.2</td>
</tr>
<tr>
<td>UK</td>
<td>2.9</td>
<td>-4.1</td>
<td>4.4</td>
<td>-5.7</td>
<td>2.4</td>
<td>-3.5</td>
</tr>
</tbody>
</table>
quantities of durables. Such reallocations seem to make some sense – of course provided that (1) production of durables is relatively capital-intensive (while that of non-durables is labour-intensive) and that (2) the rich EU countries are relatively capital-abundant (while the poorer ones are relatively labour-abundant).

7. Price equalization affects the PPP/ER ratios

The solutions to (1)–(2) allow the computation of the ratios

\[ \frac{PPP_{all\ consumption}}{PPP_{tradable\ goods}} \]

which represent pure theory’s equivalents of the PPP/ER ratios.

Table 5
Pre- and post-trade PPP/ER ratios for selected years

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>pre-trade</td>
<td>post-trade</td>
<td>pre-trade</td>
</tr>
<tr>
<td>Belgium</td>
<td>1.0271</td>
<td>1.0260</td>
<td>1.0208</td>
</tr>
<tr>
<td>Bulgaria</td>
<td>0.7125</td>
<td>0.7295</td>
<td>0.6959</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>0.7490</td>
<td>0.7667</td>
<td>0.7784</td>
</tr>
<tr>
<td>Denmark</td>
<td>1.0325</td>
<td>1.0312</td>
<td>1.0194</td>
</tr>
<tr>
<td>Germany</td>
<td>1.0092</td>
<td>1.0087</td>
<td>0.9759</td>
</tr>
<tr>
<td>Estonia</td>
<td>0.8564</td>
<td>0.8582</td>
<td>0.9214</td>
</tr>
<tr>
<td>Ireland</td>
<td>1.0556</td>
<td>1.0541</td>
<td>1.0583</td>
</tr>
<tr>
<td>Greece</td>
<td>0.9700</td>
<td>0.9738</td>
<td>0.9704</td>
</tr>
<tr>
<td>Spain</td>
<td>0.9943</td>
<td>0.9994</td>
<td>1.0189</td>
</tr>
<tr>
<td>France</td>
<td>1.0665</td>
<td>1.0636</td>
<td>1.0864</td>
</tr>
<tr>
<td>Italy</td>
<td>0.9714</td>
<td>0.9717</td>
<td>0.9948</td>
</tr>
<tr>
<td>Cyprus</td>
<td>0.8881</td>
<td>0.8922</td>
<td>0.8943</td>
</tr>
<tr>
<td>Latvia</td>
<td>0.8256</td>
<td>0.8473</td>
<td>0.8732</td>
</tr>
<tr>
<td>Lithuania</td>
<td>0.7887</td>
<td>0.8125</td>
<td>0.8267</td>
</tr>
<tr>
<td>Hungary</td>
<td>0.8105</td>
<td>0.8178</td>
<td>0.8144</td>
</tr>
<tr>
<td>Holland</td>
<td>1.0197</td>
<td>1.0197</td>
<td>1.0195</td>
</tr>
<tr>
<td>Austria</td>
<td><strong>0.9828</strong></td>
<td><strong>0.9826</strong></td>
<td>0.9968</td>
</tr>
<tr>
<td>Poland</td>
<td>0.8436</td>
<td>0.8742</td>
<td>0.8040</td>
</tr>
<tr>
<td>Portugal</td>
<td><strong>0.9140</strong></td>
<td><strong>0.9128</strong></td>
<td>0.9059</td>
</tr>
<tr>
<td>Romania</td>
<td>0.7947</td>
<td>0.8064</td>
<td>0.8271</td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.8806</td>
<td>0.8847</td>
<td>0.8902</td>
</tr>
<tr>
<td>Slovakia</td>
<td>0.7503</td>
<td>0.7752</td>
<td>0.7746</td>
</tr>
<tr>
<td>Finland</td>
<td>1.0640</td>
<td>1.0624</td>
<td>1.0450</td>
</tr>
<tr>
<td>Sweden</td>
<td>1.0155</td>
<td>1.0129</td>
<td>1.0141</td>
</tr>
<tr>
<td>UK</td>
<td><strong>1.0197</strong></td>
<td><strong>1.0244</strong></td>
<td><strong>1.0393</strong></td>
</tr>
</tbody>
</table>
Table 5 reports these ratios for selected years, together with the same ratio calculated for the original data (‘pre-trade’).

As can be seen, price-equalizing trade generally moves the PPP/ER ratios closer to unity. However, in each year there are countries whose PPP/ER ratios would – under price equalization – move away from unity.\textsuperscript{14} In 2003 these were Austria, Portugal and the UK; in 2007 Denmark, Estonia, Ireland, Spain and the UK; in 2011 Denmark, Germany, Estonia, Ireland, Portugal and the UK.

Concluding, it turns out that in a perfectly neoclassical world free (and costless) trade may be capable of increasing the gaps between purchasing power parities and exchange rates. Persistent, or even increasing, PPP-ER gaps do not disqualify the traditional neoclassical approach to international trade (and do not, per se, prove the superiority of ‘new’ trade theories).

8. Concluding remarks

It may be intuitively obvious that trade among nations – if conducted under idealized conditions assumed in pure theory – should reduce the discrepancies between exchange rates and purchasing power parities. However, this intuition would only be correct if the equalization of the prices of tradable goods (and the associated trade-induced adjustments in the volumes of consumption of tradable goods) were to leave the prices of non-tradable services unchanged.

This note has shown that in the general equilibrium context such a situation need not obtain. The changing relative prices of tradable goods may affect the demand for services. Moreover, the changes in the composition and volumes of consumption of tradable goods imply a change in the level of real income. That may also affect the demand for non-tradable services. To balance the demand for services with its fixed (available) supply, the domestic prices of services may have to change in individual countries engaging in trade. The directions and magnitudes of these changes may be different in individual countries, depending both on the pre-trade availabilities (and prices) prevailing in the given country (and in all of its partners) and on the kinds of preferences underlying ‘representative consumer’ systems of demand functions of these countries. As shown in Table 5, the general equilibrium adjustments may be such as to push the ratios identified with PPP/ER away from unity, at least in some countries. It is perhaps worth adding that such ‘perverse’ effects are possible irrespectively of the form of consumer preferences. In Podkaminer (1999) such effects were shown to be possible also under simple Cobb-Douglas and linear expenditure (Stone-Geary) preferences.

The persistence of the PPP/ER discrepancies, also under growing global integration through international trade, should not be considered puzzling. Such persistence – and instances of the discrepancies widening despite the ongoing integration – could be a normal equilibrium phenomenon even under highly idealized conditions. Needless to say, the imperfections (such as the presence of trade costs or oligopolistic practices) not considered in the pure trade theory

\textsuperscript{14} Changes in the ratios seem small. This may be due to the fact that the whole analysis works with only three broad aggregate consumption items (see also footnote 4). Arguably, an analysis distinguishing more items (both tradable and non-tradable) could suggest more pronounced changes in the PPP/ER ratios. But such a disaggregate analysis would require the estimation of a cross-country system of demand functions for many consumption items – a formidable task in itself.
can only magnify these discrepancies, and make them even more persistent. The progressing liberalization of trade (also in many service categories) and further advances in transportation/communication technologies may accelerate equalization of prices of traded goods and services. But the PPP-ER gaps may not only persist, but occasionally can also increase.

One final remark is now in order. The general exchange equilibrium model considered above forced the relative prices of tradable goods in the trading countries to be uniform – simulating the operation of the law of one price. In reality the relative prices of tradable goods are persistently dispersed. However, it may be observed that prices of tradable goods generally considered (and in particular the prices entering PPP calculations) are retail prices prevailing domestically. As such, they contain a large share of inputs of local non-tradable services such as (non-tradable) distribution services. This fact was noticed and studied by e.g. Burstein, Neves and Rebelo (2003) and McDonald and Ricci (2007).

Of course prices of local non-tradable services influencing the prices of tradable goods can differ across countries for very many reasons (such as differences in market imperfections, consumer preferences, or production technologies). However, as shown in this paper, even under ideal conditions (assuming away any imperfections, differences in tastes etc.) the recorded prices of non-tradable services may play various tricks. In particular, it cannot be ruled out that prices of tradable goods net of costs of domestic services (which are unobservable or at least not observed) might actually tend to obey the law of one price. But the recorded (gross) prices of tradable goods would not need to show this tendency even under free and competitive foreign trade – precisely because such trade could widen the cross-country dispersion of the prices of services.

References


Acknowledgements

The author is grateful to two anonymous referees and to Neil Foster for helpful suggestions on the earlier draft of this article. Financial support from the Austrian National Bank (Jubilee fund project No. 12946) is gratefully acknowledged.