Consumption function for Poland. Is life cycle hypothesis legitimate?

Katarzyna Leszkiewicz-Kędzior*, Władysław Welfe#

Submitted: 5 March 2012. Accepted: 12 July 2012.

Abstract

In industrialized countries the rising percentage of households behaves rationally. It is represented by the life cycle hypothesis of determining their incomes. This concept was taken into account in many macroeconometric multicountry models. The W models of the Polish economy assumed the number of such households is small and consequently this approach was neglected.

The paper presents the results of a research aimed at empirical testing whether the share of “rational” households in Poland was small and their majority was income constrained. We calculated the expected life cycle income on the basis of information on the structure of employment by age. The empirical study was conducted using annual data for the years 1970–2008. The obtained results show that the share of “rational” households was below 10% in the long run. But in the short run they showed a tendency of increasing up the share to more than 20%.

Keywords: consumption function, life cycle hypothesis, households’ real disposable income, households’ wealth

JEL: C51, E21

* PhD student at Chair of Econometric Models and Forecasts, University of Lodz; e-mail: k.leszkiewicz@wp.pl.
# Professor at Chair of Econometric Models and Forecasts, University of Lodz.
1. Introduction

The consumption functions are one of the basic components of the models of national economies being the core of households’ sectors. They are introduced into the macro models in various forms. In fact, two approaches can be distinguished: the traditional one where the main role is attributed to the current disposable income of households and the other in which expectations are crucial to economic decision making, following the life cycle hypothesis developed by Modigliani (1966).

Over the years, in the multicountry models built by international institutions the latter approach prevailed, whereas in the W models of the Polish economy the former one was used. The reason for this difference is that the economic situation of Polish households was unstable in the past.

The main purpose of the study is to verify whether the life cycle hypothesis is legitimate in Poland and if so to estimate the share of “rational” households. Because of close relationship between private consumption and the non-observable life cycle income, postulated by the aforementioned hypothesis, much attention is given to the calculation of this macrocategory. We followed the methodology developed in the European Community QUEST model. The inference process utilizes the Engle-Granger two step procedure with sample period consisting of annual data for the years 1970–2008.

The outline of this article is as follows. The second part presents the development of specifications of the macroeconomic consumption function. The third part contains discussion on the implications of the life cycle hypothesis in macro models. In the last part, the empirical results for Poland are reported.

2. Review of research on the consumption function

Below a review of development of macroeconomic consumption function in macroeconometric models over the last years is presented.

In the early macromodels the consumption functions explained the global consumption of households $C_t$. Aggregate consumption was mostly identified by the total real consumer expenditures, despite the fact that the value of services flows of consumer goods would be equal to the expenditures only in the case of non-durables and services.

The above relationships had a Keynesian origin. The major explanatory variable was after-tax net real disposable income $Y_t$. To allow for competitive role of savings the real interest rate $R_t$ (either short- or long-term) was introduced as additional explanatory variable. It can be represented by the following long-run equation:

$$\ln C_t^* = \alpha_0 + \alpha_1 \ln Y_t + \alpha_2 (1 + R_t) + \xi_t$$

(1)

The real incomes were often complemented by adding the real net credits to allow for this additional source of purchasing funds. The above specification represents the long-run consumer demand function. Hence, it was frequently assumed that the elasticity of consumption w.r. to the real disposable income can be calibrated at the level equal to one ($\alpha_1 = 1$), which expressed the belief that in the long run the savings ratio stabilizes (but only if the real interest rate is stationary).
Relatively early it has been observed inertia in households behaviour. This habit persistence, following the proposal of the Canadian econometrician Brown (1952), can be represented by the lagged consumption $C_{t-1}$. This modification is expressed by the following equation:

$$\ln C_t^\ast = \alpha_0 + \gamma_1 \ln C_{t-1} + \alpha_1 \ln Y_t + \alpha_2 (1 + R_t) + \xi_t$$  \hspace{1cm} (2)

Assuming $C_t = C_{t-1}$, the long-term elasticities can be calculated. The elasticity w.r. to $Y_t$ equals $\alpha_1 / (1 - \gamma_1)$.

As an example let us quote the results obtained for Poland (based on the sample covering the years 1960–1998): the estimate of the autoregression coefficient was equal to $\gamma_1 = 0.53$, whereas the short-run elasticity w.r. to real disposable income was $\alpha_1 = 0.40$, and the long-term elasticity equalled 0.85 (see Welfe 2001, p. 61).

The habit formation coefficients obtained in the 21st century gave results ranging from 0.6 to 0.8 for the UK, US and the euro area (in the Bank of England Quarterly Model, BEQM) (see Harrison et al. 2005, pp. 109–110).

Let us mention that the hypothesis postulating that the demand depends on the real interest rate occurred unsatisfactorily because parameter estimates showed most of the time unacceptable positive values. This induced many model builders to assume that the money illusion takes place in the consumer decisions. Hence, the nominal interest rates were used instead (see e.g. Fair 2004). Sometimes the values of $\alpha_2$ parameter were calibrated (see Beneš et al. 2005) or the interest rate was omitted at all.

In the 1990s the model builders started to distinguish between the long-term and short-term demand functions. The short-term equations were dynamic and the error correction models (ECM) were used. The point of departure was the static long-term equation (1). The short-term equation had the standard form:

$$\Delta \ln C_t = \alpha_0 + \beta_1 (\ln C_{t-1} - \ln C_{t-1}^\ast) + \beta_2 \Delta \ln Y_t + \beta_3 \Delta (1 + R_t) + \xi_t$$  \hspace{1cm} (3)

The specific property of the above equation, which for many years has been used in the W-macromodels, is that the consumer demand is dependent on the current real disposable income only, i.e. that the households are liquidity constrained. In the 1980s in the United Kingdom the attention was paid to the so far ignored impact of real personal wealth of households $W_p$, modifying the households behaviour. Hendry and Ungarn-Sternberg (1981) introduced the financial wealth $WL_{p}$, and in the mid 1980s the necessity was recognized to introduce the physical wealth $WN_p$ (housing wealth) as an additional variable. The introduction of $WL_p$ ensured the transmission of shocks of the financial sector to the real sector.

It has been argued by many researchers that introducing personal wealth as additional explanatory variable that is competitive to the real personal income induces imposition of the homogeneity restriction. The sum of elasticities of consumption w.r. to real disposable income and real personal wealth should equal one. The following long-term equation meets the homogeneity restriction:

$$\ln C_t^\ast = \alpha_0 + \alpha_1 \ln Y_t + (1 - \alpha_1) \ln W_p + \alpha_2 (1 + R_t) + \xi_t$$  \hspace{1cm} (4)
This function is still used in many macroeconometric models, including the W8 models of the Polish economy.

The parameters of the dynamic, short-run equations are typically (since Davidson et al. 1978) estimated using the error correction model (ECM). It is because the variables in (4) are most frequently non-stationary, but their first differences are usually stationary. This supposition is subject to relevant testing.

This yields a conventional short-term equation, in which we assumed a lag of one period (one quarter in quarterly models):

$$\Delta \ln C_t = \beta_0 + \beta_1 (\ln C_{t-1} - \ln C^*_t) + \beta_2 \Delta \ln Y_t + (1 - \beta_2) \Delta \ln W_t + \beta_3 \Delta (1 + R_t) + \xi_t \quad (5)$$

where $C^*$ is long-term consumer demand function as in (4).

Further dynamization of the consumer demand function implied the introduction of expectations of explanatory variables, including rational expectations. It especially applied to the UK models, but also to the US economy models built at the FRB. Let us present as an example a simple specification of the short-term equation used in the UK NIGEM model (see Barrell et al. 2004). To equation (5) the expected consumption with one period lag was introduced as an additional explanatory variable:

$$\Delta \ln C_t = \mu [\ln C_{t-1} - \alpha \ln Y_{t-1} - (1 - \alpha) \ln W_{t-1}] + \delta \Delta \ln C_{t-1} + \varepsilon_t \quad (6)$$

where $\delta$ is the rate of time preference.

In this forward looking equation the parameter was calibrated at the level equal to $\delta = 0.97$. The estimates of the elasticities $\alpha$ were obtained at the level 0.83 for France, Germany and Italy and at 0.86 for the United Kingdom.

In the models of several countries additional explanatory variables were added. These were the unemployment rate; its increase would postpone the current consumption (models of Belgium and Lithuania), or the level of professional activity or even the output-gap (GLOBAL-FRB for USA).

3. The permanent income and life cycle hypothesis

This part contains the discussion on the properties of the theory of the life cycle hypothesis and its applications in modelling consumption in macro models.

In the 80s the specification of consumer demand functions in many models seriously changed, following the principles of the neoclassical theory of consumer behaviour. The permanent income hypothesis was formulated by Friedman (1957), assuming that the unobservable permanent income is an adequate determinant of consumption. The life cycle hypothesis of savings was proposed by Modigliani, Ando (1957), Ando, Modigliani (1963) and Modigliani (1966), being based on the results of some previous utility analysis (Modigliani, Brumberg 1954; see also Modigliani 1975).
The permanent income hypothesis postulated that the consumption can be split into permanent consumption dependent on permanent income and transitory consumption related to the transitory income composed of unstable, incidental elements. This unobservable variable might be approximated – following the suggestion of Muellbauer and Lattimore (1995) – by the weighted average of the real disposable income and personal wealth. In fact, it would mean a return to the initial specification (4).

More general application found the life cycle hypothesis using the concept formulated by Blanchard (1985). It makes use of the model of overlapping generations that assumes that the consumers optimize the consumption over their life cycle given a fixed time horizon (which follows the predetermined probability of death). This model refers to the rational behaviour of households. Let us mention that an earlier hypothesis of intertemporal optimalization was formulated by Hall (1978) yielding an Euler equation.

Following Cambell and Mankiw (1991) it had to be recognized, however, that a certain proportion of households is liquidity (credit) constrained, i.e. their decisions are based on the current real disposable income. This share was estimated in the MULTIMOD model at 44% on average, in the model QUEST at 30%, being seriously different for particular countries. For the USA it was estimated at only 10% in FRB/US model (see Levin, Rogers, Tryon 1997).

In theory the rational household (consumer) is assumed to maximize its discounted utility over the life cycle, given its predetermined, expected life. The aggregation leads to the following long-run consumption function, where the consumption is related to the real wealth $W_t$:

$$C_t^n = \alpha W_t$$

(7)

where: $W_t = V_{t-1} + H_t$, $H_t$ is the present expected human real wealth and $V_t$ is the real financial wealth.

Several authors are looking for additional implications of the life cycle hypothesis. Let us mention the studies on the impact of the consumption total wealth ratio, i.e. of $\alpha$ on the expected stock returns (see e.g. Lettau, Ludvigson 2001; Zachlod-Jelec 2010). There are also microeconomic and financial applications (see e.g. Carroll 1994; Huggett 1996).

The expected human real wealth can be calculated from the equation:

$$H_t = \int_{t=0}^{T} (Y_t)e^{-(R + n)\tau} dt$$

(8)

where:

- $p$ – the probability of death,
- $n$ – the population rate of growth,
- $Y_t$ – the real net labour income,
- $T$ – age at which working life and life cycle end.

This equation can be approximated as follows:

$$H_t = \sum_{t=0}^{T} \left( \frac{1 - p}{1 - R} \right) Y_t$$

(9)
The real financial wealth gained a broad notion. It should include the market value of enterprises, government debt and net foreign assets. Its justification is – following Masson, Symansky and Meredith (1990) – the assumption that the household sector imposes full control over the domestic financial assets. Several authors treat these components separately, looking for different marginal propensities to consume.

The parameter \( \alpha \) representing the long-run marginal propensity to consume can be treated as a function of relative risk aversion, of the rate of time preference \( \delta \), being dependent on the intertemporal elasticity of substitution, on the real interest rate \( r \) and the probability of death \( p \):

\[
\alpha = 1 - (1 - p)(1 - \delta)(1 - R) + \ldots
\]  

(10)

The equation to be estimated is obtained adding the component determined by the expected wealth and the component related to the liquidity constrained households, being dependent on current real disposable income:

\[
C_t = C_t^w + f(Y_t)
\]  

(11)

In practice, the double-log approximation is most frequently used and distinction drawn between the real human and financial wealth:

\[
\ln C_t = \alpha_0 + \alpha_1 \ln H_t + (1 - \alpha_1)\ln V_t + \alpha_2 \ln Y_t + \xi_t
\]  

(12)

In applications it was assumed in general that \( p = 0.02 \), which is equivalent to the assumption that the number of expected years of life equals 50. The rate of time preference was assumed to be 0.009 on the average (from 0.005 for Japan to 0.01 for the US following the QUEST model). The elasticity of intertemporal substitution showed a wide range, from low 0.2 for the UK, via 0.35 for the US and to 0.6 for the US according to Smets and Wouters (2004) as reported in the BEQM model description (see Harrison et al. 2005, p. 109).

The calculation of the financial wealth is pretty complicated. Nevertheless, the above concepts were used in the construction of consumption functions in many multicountry models like MULTIMOD, QUEST or country models in Belgium, Finland (BFO5), Spain and also in the recently built DSGE models.

In the W8 models of the Polish economy it was assumed that the majority of Polish households is income constrained. It is because of low stability of households’ environment in the past. The new generations of households involved in planning their long-life behaviour was assumed to be very small but growing. In the previous W8 models they were ignored. In order to determine whether this was appropriate, an empirical test is necessary. Hence, we decided to specify the consumption function based on the life cycle hypothesis. This is the subject of the next section.
4. Empirical analysis

The empirical analysis consisted of two stages: calculating the non-observable macrocategory called life cycle income, understood by the life cycle hypothesis as the consumers’ whole life income, and estimating the consumption function for Poland in order to find out what is the share of households behaving rationally. The study uses annual time series from official statistical sources for the period 1970–2008 with an exception of one variable referring to the financial wealth starting in 1991 (see Figures 1–6). The data was transformed into real terms using the appropriate deflators with the base period in 1995.

To verify the life cycle hypothesis in Poland, the consumption function following (7) and (10) was used. Under the life cycle hypothesis the consumption of the rational households can be presented as follows:

\[ C_i^w = (\delta + p)(LCI_i + FW_i) \]  \hspace{1cm} (13)

where \( LCI \) denotes life cycle income defined as the sum of current and future expected net income, \( FW \) is real financial wealth, \( \delta \) is the rate of time preference and \( p \) is the probability of death. We follow here the developments presented in the QUEST model (see Roeger, in’t Veld 1997).

Many results of empirical studies of consumer behaviour, including the \( W \) models for Polish economy, confirm, however, that consumption depends to some extent on real current disposable income because of liquidity constrained households. Therefore, following Campbell and Mankiw (1991), the consumption function given by formula (13) was extended as follows:

\[ C_i = (1 - \lambda)(\delta + p)(LCI_i + FW_i) + \lambda Y_i \]  \hspace{1cm} (14)

where the value of \( (1 - \lambda) \) parameter is the subject of interest and refers to a share of households behaving rationally in making decisions about consumption.

In order to perform the estimation of the above consumption function, the assumptions about the values of \( \delta \) and \( p \) parameters had to be made. Based on earlier studies, we assumed that the probability of death is equal to 2% which means that the life expectancy is equal to 50. But the rate of time preference was assumed to be equal to 0.009 being the average of the QUEST model.

As the life cycle income, being the key variable in the proposed specification of consumption function (14), is non-observable, it was necessary to calculate its values. It was assumed that this macrocategory can be determined using information on the structure of employment by age, but additional assumptions were required about the values of nominal interest rate and the growth rate of wage bill outside the sample period. Therefore, it was assumed that the nominal interest rate is equal to 0.025, which is equivalent to the assumption that the real interest rate equals 0, if the inflation rate takes the value of the inflation target. The growth rate of the real wage bill was assumed to be equal to 0.03, which means that the growth rate of the nominal wage bill is 5.5%. And the last assumption was done that the initial age of people earning and making decisions...
about consumption is 20 years, which in combination with the assumption that \( p = 0.02 \) means that people being between 20 and 69 were considered.

The proposed method involved the following three steps. First, using the distribution of wages by age given in Figure 7, the average wages for each of the age groups \( W_k \) (\( k = 20, \ldots, 69 \)) had to be determined. Next, the wage bill of age groups of 20–69 years \( WB_k \) was estimated according to formula (15):

\[
WB_k = \sum_{k=26}^{69} u_k N_k W_k
\]

where \( u_k \) is the share of a given age group in the total employment and \( N \) is total employment.

Finally, the life cycle income \( LCI_k \) was calculated based on the formula (16), where the current and future discounted incomes are added up together and divided by the number of expected years of life:

\[
LCI_k = \frac{WB_k + \frac{WB_{k+1}}{1+r_{i-1}} + \sum_{i=2}^{49} \frac{WB_{i+1}}{(1+r_{i-1}) \ldots (1+r_{i-1})}}{50}
\]

where \( r \) denotes the nominal interest rate.

Figure 8 presents the estimated life cycle income in the period 1970–2008. It can be seen that future expected net income was increasing all the time, but after 1990 the slope decreased, and after 2002 it increased again. The reason for this change may be the fact that during calculation process from the observation for 1990 the assumed values received higher weights than the real values. Therefore, it is worth noting that estimation results are sensitive to changes in assumptions. For example, an increase in the wage bill or a decrease in the interest rate would be followed by additional increase in the life cycle income after 1990. It seems, however, that the assumed values of the above variables are the most realistic.

After the life cycle income had been calculated, the parameters of consumption function (14) were estimated using the two-step Engle-Granger procedure (see Engle, Granger 1987). The order of integration of the variables was determined first based on the standard unit root tests, ADF and KPSS, and their results explicitly showed that all variables were integrated of order one (see Table 1).

The estimation procedure yielded the following estimates of the consumption function’s long-run parameters (the values of t-Student statistics are given in the parantheses):

\[
\ln C_t = 0.807 + 0.070(\delta + p) \ln(LCI_t + FW_t) + 0.930 \ln Y + \text{relevant dummies}
\]

The F test for linear restrictions, at a borderline level of significance of 0.115 (\( F = 2.627 \)), shows no grounds for rejecting the homogeneity restriction following the life cycle hypothesis. The estimation results show that Polish households’ decisions were mainly made on the basis of current
personal incomes, the estimate of $\lambda$ parameter is equal to 93%. Nevertheless, the hypothesis that there is a percentage of households behaving rationally was confirmed and the share was estimated at 7%.

The analysis of long-run residuals yielded positive results (see Table 2). At the standard level of significance of 0.05 there are no reasons for rejecting the hypothesis that the residuals are homogeneous, normally distributed and not autocorrelated. The result of the CUSUM test for parameter stability is also satisfactory. What is important, the ADF test for residuals confirmed its stationarity, which means that the variables are cointegrated and a short-run relationship between them can be expressed as a standard form of error correction model (ECM).

The estimation results of the short-run dynamics parameters are as follows:

$$\Delta \ln C_t = -0.819 ECT_{t-1} + 0.211(\delta + p) \Delta \ln(LCI_t + FW_t) + 0.789 \ln Y + \text{relevant dummies} \quad (18)$$

At a borderline level of significance of 0.804 ($F = 0.063$) the homogeneity restriction cannot be rejected. Hence, it was confirmed that the life cycle hypothesis holds true also in the short run. What is more, the estimate of $\lambda$ parameter decreased by 14 pp, from 93% to 79%, which means that the share of the rational households had tripled to 21%. Such significance increase in that value gives the grounds to suppose that in the Polish economy the growing tendency among households to behave rationally takes place. The results of the residual analysis are also positive in the case of short-run equation (see Table 3).

It is worth noting that a sensitivity analysis to changes in assumptions was performed to assess to what extent the obtained results depend on them. The analysis showed that the final findings were robust to variation in assumptions. Differences in estimates of the share of “rational” households were around 1 pp, which allows to answer the title question in the affirmative.

5. Conclusions

Polish households used to make their decisions about consumption in the conditions of instability of economic and social environment. Therefore, the results showing that households’ decisions were made on the basis of current personal incomes are not surprising. The hypothesis that there is a certain percentage of households consisting of young couples with high economic activity that behave rationally was confirmed. The share was estimated at an average of 7% and 21% in the long- and short-run, respectively. Therefore, it can be concluded that the life cycle hypothesis in Poland is legitimate.

The study is worth repeating for an extended sample to verify if the share of households behaving rationally increases in time.

In the W8 model we decided to include two alternative versions of consumption function relying on the results of the research. We expect that in the long term the share of rational households will be increasing, which implies the use of equations based on the life cycle hypothesis.
References


Appendix

Table 1
Inference on the order of integration

<table>
<thead>
<tr>
<th>Variable</th>
<th>Set of hypothesis</th>
<th>Test ADF</th>
<th>Test KPSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔlnC</td>
<td>ADF: I(2) vs. I(1)</td>
<td>-4.606 I(1)</td>
<td>0.155 I(1)</td>
</tr>
<tr>
<td>Δln(LCI + FW)</td>
<td>KPSS: I(1) vs. I(2)</td>
<td>-5.660 I(1)</td>
<td>0.110 I(1)</td>
</tr>
<tr>
<td>ΔlnY</td>
<td>ADF: I(1) vs. I(0)</td>
<td>-5.463 I(1)</td>
<td>0.163 I(1)</td>
</tr>
<tr>
<td>lnC</td>
<td>ADF: I(1) vs. I(0)</td>
<td>-1.022 I(1)</td>
<td>1.733 I(1)</td>
</tr>
<tr>
<td>ln(LCI + FW)</td>
<td>KPSS: I(0) vs. I(1)</td>
<td>0.330 I(1)</td>
<td>1.949 I(1)</td>
</tr>
<tr>
<td>lnY</td>
<td></td>
<td>-1.747 I(1)</td>
<td>1.806 I(1)</td>
</tr>
</tbody>
</table>

Note: inference performed at a 0.05 level of significance. The only deterministic variable used in verifying the order of integration of the variables is constant term. The critical value of the ADF test is 2.86 (see Davidson, MacKinnon 1993), and 0.463 for the KPSS test (see Kwiatkowski et al. 1992).

Table 2
The residual analysis (long-run equation)

<table>
<thead>
<tr>
<th>Test</th>
<th>Test statistics</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>White’s test for heteroskedasticity</td>
<td>6.047</td>
<td>0.642</td>
</tr>
<tr>
<td>LM test for autocorrelation of order 1</td>
<td>1.424</td>
<td>0.241</td>
</tr>
<tr>
<td>Test for normality of residuals</td>
<td>3.707</td>
<td>0.157</td>
</tr>
<tr>
<td>CUSUM test for parameter stability</td>
<td>0.920</td>
<td>0.364</td>
</tr>
<tr>
<td>ADF test for residuals</td>
<td>-4.453</td>
<td>0.045</td>
</tr>
</tbody>
</table>
Table 3
The residual analysis (short-run equation)

<table>
<thead>
<tr>
<th>Test</th>
<th>Test statistics</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>White’s test for heteroskedasticity</td>
<td>7.296</td>
<td>0.775</td>
</tr>
<tr>
<td>LM test for autocorrelation of order 1</td>
<td>0.405</td>
<td>0.529</td>
</tr>
<tr>
<td>Test for normality of residuals</td>
<td>2.561</td>
<td>0.278</td>
</tr>
<tr>
<td>CUSUM test for parameter stability</td>
<td>-0.106</td>
<td>0.916</td>
</tr>
</tbody>
</table>

Figure 1
Real private consumption

Figure 2
Real disposable income
Figure 3
Real financial wealth

![Graph showing real financial wealth over time.](image)

Figure 4
Average nominal gross wages

![Graph showing average nominal gross wages over time.](image)
Figure 5
Reference nominal interest rate

Figure 6
Total employment
Figure 7
The distribution of wages by age (relative to the age group of 15 years)


Figure 8
The estimated life cycle income in the period 1970–2008